Monday, Oct 29, 2018

NOTES

- Warm-UP
- Review Permutations & Introduce Combinations (notes cont.)
- Intro to Pascal's Triangle & Binomial Theorem
- Blind LUCK Quiz
- HW Practice

## Warm-UP

1. How many different ways can you answer a 10 question, true/false quiz?
2. How many different ways can you answer a 10 question, multiple-choice quiz? (*assuming there are 4 choices for each question*)
3. What is the typical result of a binomial squared?

Warm-UP Answers

1) How many different ways can you answer a 10 question, true/false quiz?

   There are **2 options** for each question, so it's $2^{10} = 1024$

*2)* How many different ways can you answer a 10 question, multiple-choice quiz? (*assuming there are 4 choices for each)*

   There are **4 options** for each question, so $4^{10} = 1{,}048{,}576$

*3) What is the typical result of a binomial squared?*
   *A* ***perfect square trinomial****, except when…it's* ***complex***

$(3 - i)^2 = (3 - i)(3 - i) = 9 - 6i + \boldsymbol{i^2}$
$= 9 - 6i + \ -1$

$$so\ (3 - i)^2 = 8 - 6i$$

$\ldots a\ perfect\ square, but\ \textbf{not}\ a\ trinomial$

Thursday, Oct 25, 2018

BLIND Luck Quiz – ☺

Please answer the following True/False questions about probability:

1. Q1
2. Q 2
3. Q 3
4. Q 4
5. Q 5
6. Q 6
7. Q 7
8. Q 8
9. Q 9
10. Q 10

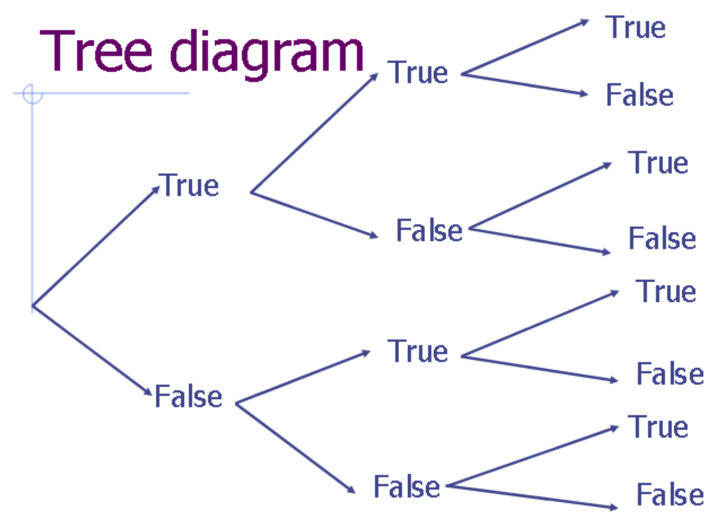
Blind LUCK Quiz!

Answer True/ False

| | T | F |
|---|---|---|
| Q1 | T | |
| Q2 | | F |
| Q3 | T | |
| Q4 | T | |
| Q5 | | F |
| Q6 | | F |
| Q7 | T | |
| Q8 | T | |
| Q9 | T | |
| Q10 | | F |

Tree Tracking

True/ False Answer Paths




Q1 2 ways

Q2 4 ways

Q3 8 ways …

… etc.

What are the number of ways to answer a 10 question, true/false quiz? 1024

Review Permutations & Introduce Combinations

(notes continued)

Consider the following: *How many ways can 3 students be chosen for a candy treat, from a group of 15?*

A **combination** is a selection of objects or events in which the order ***does not*** matter.

In general you can find the number of combinations for an event by finding the number of *permutations* and then eliminate the selections that have the **same objects**, simply in a different order.

Notation for combinations:

$_{n}C_{r}$ represents the number of ***C***ombinations of

***n*** things taken ***r*** at a time

So the number of ways you can select 3 students from a group of 15 would be:

$$_{n}C_{r} = {}_{15}C_{3}$$

Note: our Quality Core cheat sheet uses $_{k}C_{m}$ in place of $_{n}C_{r}$

$$ {}_nC_r \rightarrow {}_{15}C_3 = \frac{15 \cdot 14 \cdot 13}{3 \cdot 2 \cdot 1} \text{ or} $$

$$ {}_{15}C_3 = \frac{{}_{15}P_3}{3!} $$

*If you go bowling and knock down 6 pins, how many combinations of pins can remain?*

$$ {}_nC_r \rightarrow {}_{10}C_4 = \frac{10 \cdot 9 \cdot 8 \cdot 7}{4 \cdot 3 \cdot 2 \cdot 1} \quad \text{or} $$

$$ {}_{10}C_4 = \frac{{}_{10}P_4}{4!} = \frac{10!}{6! \cdot 4!} $$

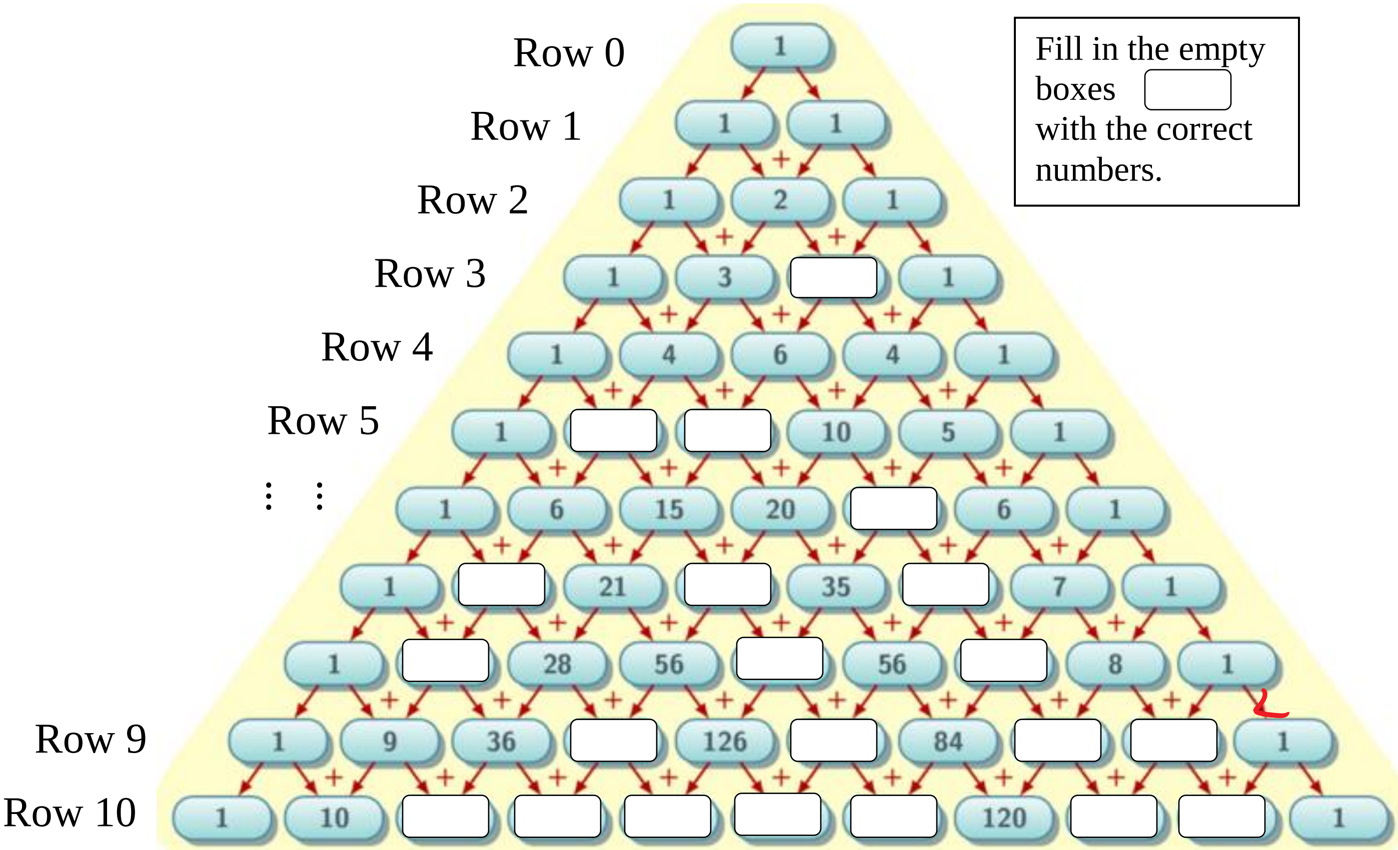
Notation for combinations in general and how to calculate them:

$$ {}_nC_r \rightarrow \; = \frac{n!}{(n-r)! \cdot r!} $$

$$ ex: \qquad {}_{25}C_5 = \frac{{}_{25}P_5}{5!} = \frac{25!}{20! \cdot 5!} $$

Name: ______________________________

## Patterns with Pascal's Triangle:




Fill in the empty boxes with the correct numbers.

Binomial coefficients relate to __________________ in that you can predict the _____________ of any binomial expansion. For example, what is the product of $(x+2y)^6$ ? Using the binomial expansion based on the binomial coefficients, this is simple. Look at _______ of Pascal's triangle, which indicates the coefficients of each term, as well as the number of terms in the final product: **7 terms**, w/ leading coeff: 1, 6, 15, 20, 15, 6, 1

$$(x+2y)^6 = 1(x^6 \cdot (2y)^0) + 6(x^5 \cdot (2y)^1) + 15(x^4 \cdot (2y)^2) + 20(x^3 \cdot (2y)^3) + \cdots$$

$\therefore\ (x+2y)^6 =$

(write your solution!)

The pattern in each ROW corresponds to the number of ____________________ that occur, or the number of ways to choose certain objects where order does not matter. The notation we use is ${}_8C_3$ or $C\binom{8}{3}$ which means choosing _____ objects out of a group of _______.

Patterns with polynomial expansion:

## Consider the coefficients

$$
\begin{aligned}
(a+b)^0 &= 1\\
(a+b)^1 &= a+b\\
(a+b)^2 &= a^2+2ab+b^2\\
(a+b)^3 &= a^3+3a^2b+3ab^2+b^3\\
(a+b)^4 &= a^4+4a^3b+6a^2b^2+4ab^3+b^4\\
(a+b)^5 &= a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5
\end{aligned}
$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

For any binomial to any power:

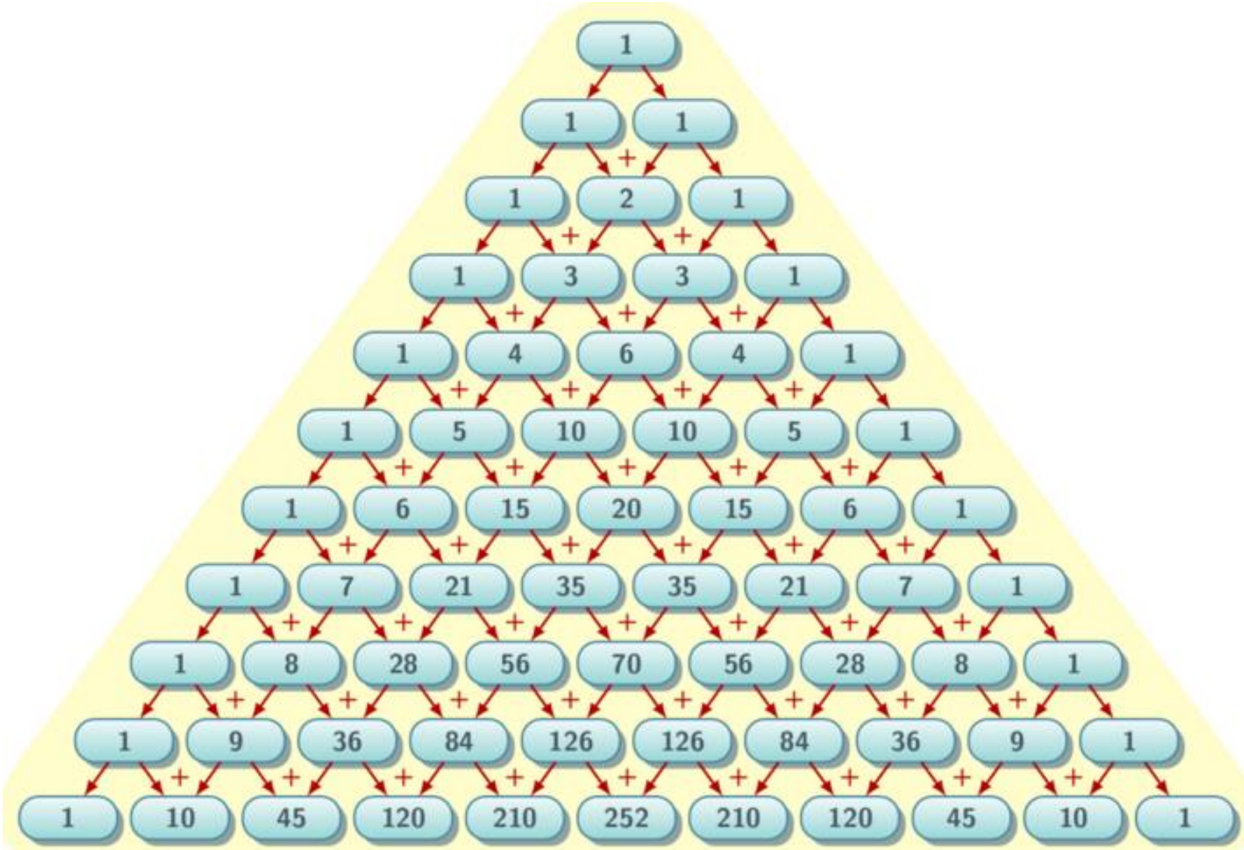
$$(a+b)^n = {}_nC_0\, a^n b^0 + {}_nC_1\, a^{n-1} b^1 + {}_nC_2\, a^{n-2} b^2 + \cdots + {}_nC_n\, a^0 b^n$$

Example:

$$(a+b)^9 = {}_9C_0\, a^9 b^0 + {}_9C_1\, a^8 b^1 + {}_9C_2\, a^7 b^2 \cdots + {}_9C_8\, a^1 b^8 + {}_9C_9\, a^0 b^9$$

Binomial Formula

$$(a+b)^n = \sum_{r=0}^{n} {}_nC_r\, a^{n-r} b^r$$




Binomial coefficients relate to **Pascal's Triangle** in that you can predict the **product** of any binomial expansion. For example, what is the product of $(x+2y)^6$ ? Using the binomial expansion based on the binomial coefficients, this is simple. Look at **ROW 6** of Pascal's triangle, which indicates the coefficients of each term, as well as the number of terms in the final product: **7 terms**, w/ leading coeff: 1, 6, 15, 20, 15, 6, 1

$$(x+2y)^6 = 1(x^6 \cdot (2y)^0) + 6(x^5 \cdot (2y)^1) + 15(x^4 \cdot (2y)^2) + 20(x^3 \cdot (2y)^3) + \cdots$$

$\therefore\ (x+2y)^6 =$ (HERE is the solution!)

$$\boldsymbol{x^6 + 12x^5y + 60x^4y^2 + 160x^3y^3 + 240x^2y^4 + 192xy^5 + 64y^6}$$

The pattern in each ROW corresponds to the number of **combinations** that occur, or the number of ways to choose certain objects where order does not matter. The notation we use is $_8C_3$ or $C\binom{8}{3}$ which means choosing **three** objects out of a group of **eight**.

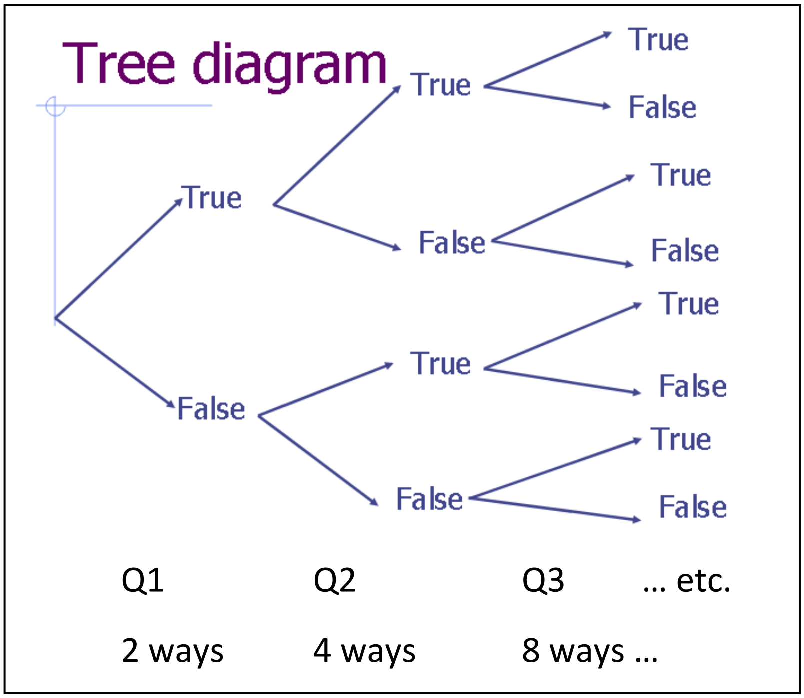
Blind LUCK Quiz!

Answer True/ False

| | T | F |
|---|---|---|
| Q1 | T | |
| Q2 | | F |
| Q3 | T | |
| Q4 | T | |
| Q5 | | F |
| Q6 | | F |
| Q7 | T | |
| Q8 | T | |
| Q9 | T | |
| Q10 | | F |

Tree Tracking

True/ False Answer Paths




What are the number of ways to answer a 10 question, true/false quiz? ______________

What are the number of way to get exactly ***x*** questions correct? (*use the table below*)

| 0 Correct | 1 Correct | 2 Correct | 3 Correct | 4 Correct | 5 Correct | 6 Correct | 7 Correct | 8 Correct | 9 Correct | 10 Correct |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 10 | 45 | . . . | | | | | | 10 | 1 |

What is the probability of getting:

a) zero correct? ___________
b) one correct? ___________
c) two correct? ___________
d) three correct? ___________
e) four correct? ___________
f) five correct? ___________
g) six correct? ___________
h) seven correct? ___________
i) eight correct? ___________
j) nine correct? ___________
k) All ten correct? ___________

What are the number of ways to answer a 10 question, true/false quiz? __**1024**____

What are the number of way to get exactly ***x*** questions correct? (*use the table below*)

| 0 Correct | 1 Correct | 2 Correct | 3 Correct | 4 Correct | 5 Correct | 6 Correct | 7 Correct | 8 Correct | 9 Correct | 10 Correct |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 10 | 45 | 120 | 210 | 252 | 210 | 120 | 45 | 10 | 1 |

What is the probability of getting:

a) zero correct? $\frac{1}{1024} = 9.765625\,\mathbf{E}\,^{-}4 \approx 0.0009765$

b) one correct? $\frac{10}{1024} = 0.0097656$

c) two correct? $\frac{45}{1024} = 0.0439453$

d) three correct? ___________

e) four correct? ___________

f) five correct? ___________

(simply follow the pattern)