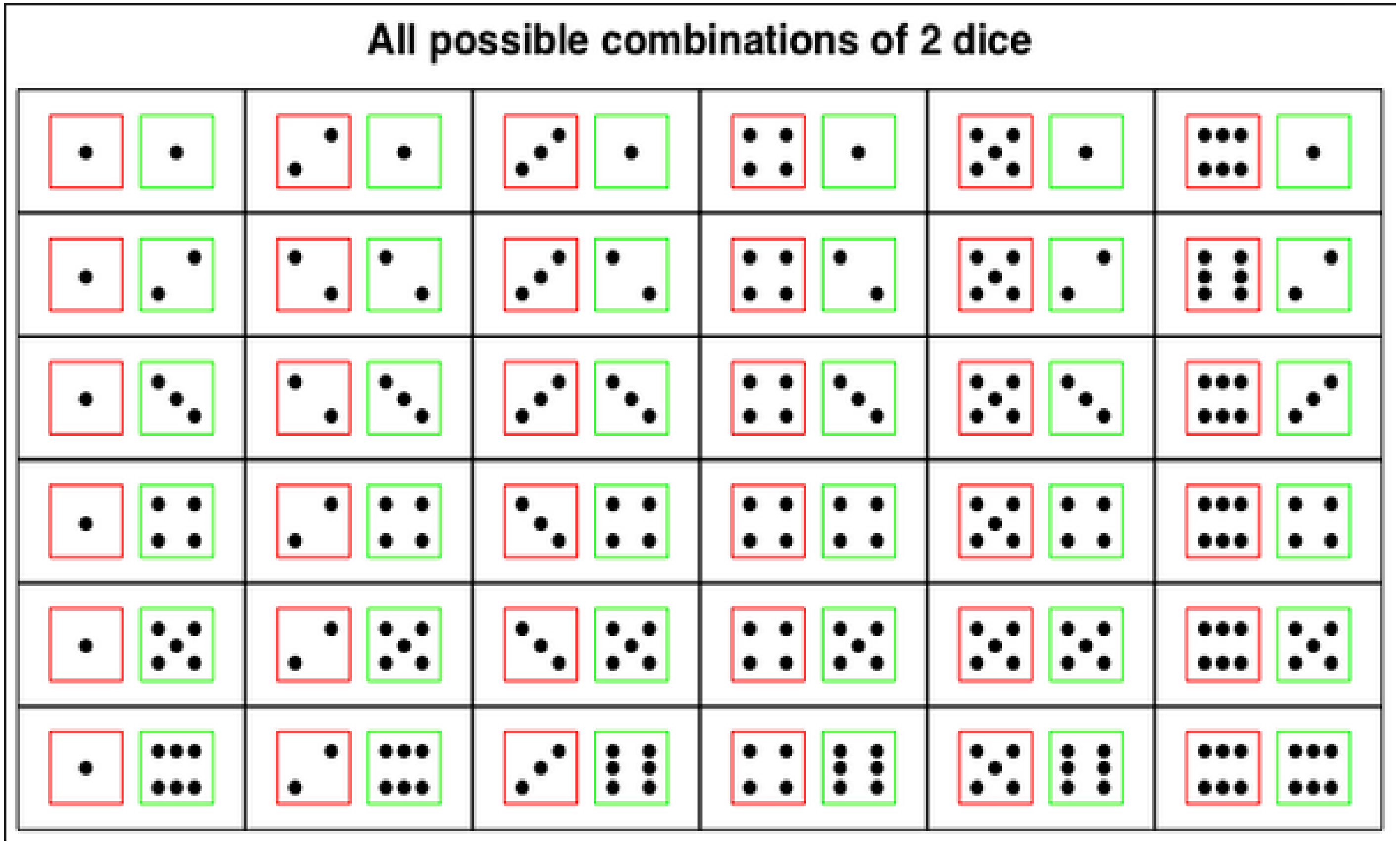
# Objectives

- Students will find the probability of an event (and the odds of an event).
- Students will understand and be able to compare: (1) Classical probability, (2) Empirical probability, and (3) Subjective probability
- Students will understand and use the vocabulary and notation associated with probability

# Vocabulary

- **Probability (P)** – is the likelihood that an event will occur.
- **Outcomes** – when you do a probability experiment, each *result* of a single trial is called an outcome.
- **Event** – is an outcome or a collection of outcomes
- **Sample Space** - A list of every possible outcome for a given condition (i.e., rolling dice, or drawing cards, etc.)

# What is the sample space for rolling 2 dice?




36 possible outcomes

# Vocabulary

- **Fundamental Counting Principle** – is used to determine the total number of ways that successive events can occur
- So, if there are *m* ways for one event to occur, and *n* ways for another event to occur, then there are $m \cdot n$ ways for both events to occur
- If you have 5 shirts, 4 pants, and 7 pairs of shoes, you can make __**140**__ outfits.

# Vocabulary

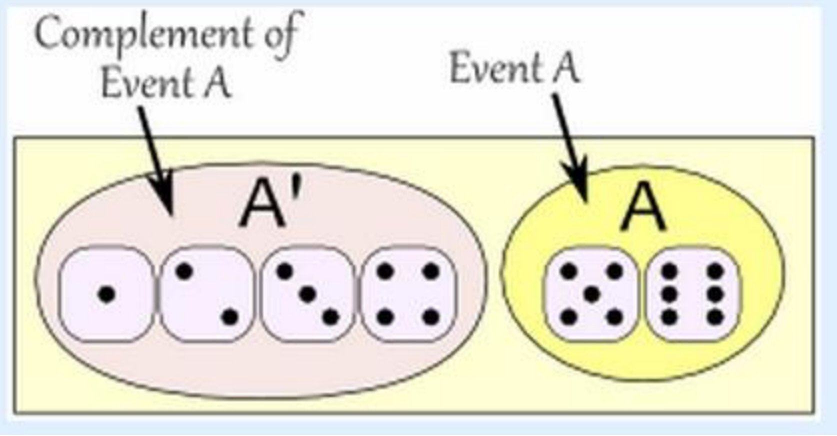
- **Complement** of an Event P(A') are all of the other outcomes ***not*** in Event A

Example: Rolling a "5" or "6"

**Event A**: {5, 6}

Number of ways it can happen: 2

Total number of outcomes: 6




$$P(A) = \frac{2}{6} = \frac{1}{3}$$

$$P(A') = \frac{4}{6} = \frac{2}{3}$$

The **Complement of Event A** is {1, 2, 3, 4}

$$P(A) + P(A') = 1$$

# Types of Probability

- There are 3 types of probability

Theoretical Probability
Experimental Probability
Subjective Probability

- Let's look at each one individually…

# Classical Probability

- Classical or *Theoretical* Probability is based upon the number of favorable outcomes divided by the total number of outcomes

**Example:**

- In the roll of a die, the probability of getting an even number is 3/6 or ½.
- Notation used:

$$P\ (\text{even}) = \frac{3}{6}\ or\ \frac{1}{2}$$

# How does that work?

- Typically a six-sided die contains the numbers 1, 2, 3, 4, 5, and 6.
- Of those numbers only 2, 4, and 6 are even.
- So, we can set up a ratio of the number of favorable outcomes divided by the total number of outcomes, which is 3/6 or 1/2

# Theoretical Probability Formula

If we denote ***A*** = *desired event*, then
Probability of this event is: P(***A***)

Theoretical Probability :

$$P(\boldsymbol{A}) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

# Example # 1

- A box contains **5 green** pens, **3 blue** pens, **8 black** pens and **4 red** pens. A pen is picked at random

- What is the probability that the pen is green?

There are **5** + **3** + **8** + **4** or **20** pens in the box

$$P(\text{green}) = \frac{\#\text{ green pens}}{\text{Total }\#\text{ of pens}} = \frac{5}{20} = \frac{1}{4}$$

# Experimental (Empirical) Probability

- As the name suggests, **Experimental** (or *empirical*) **Probability** is based upon repetitions of an actual experiment.

**Example:**

If you toss a coin 10 times and record heads for 8 trials, then the experimental probability was $P(heads) = \frac{8}{10} = \frac{4}{5}$

# Experimental Probability Formula

- Experimental Probability:

$$P(E) = \frac{\text{Number of favorable outcomes}}{\text{Total number trials}}$$

# Example #2

- In an experiment a coin is tossed **15** times. The recorded outcomes were: **6 heads** and **9 tails**. What was the experimental probability of the coin being heads?

$$P\ (\text{heads}) = \frac{\#\ \text{Heads}}{\text{Total}\ \#\ \text{Tosses}} = \frac{6}{15}$$






# Subjective Probability

- **Subjective probability** describes an individual's personal judgement about how likely a particular event is to occur. It is not based on any precise computation but is often a reasonable assessment based upon given knowledge.

- It is still expressed within the scale from 0 (impossible) to 1 (certain).

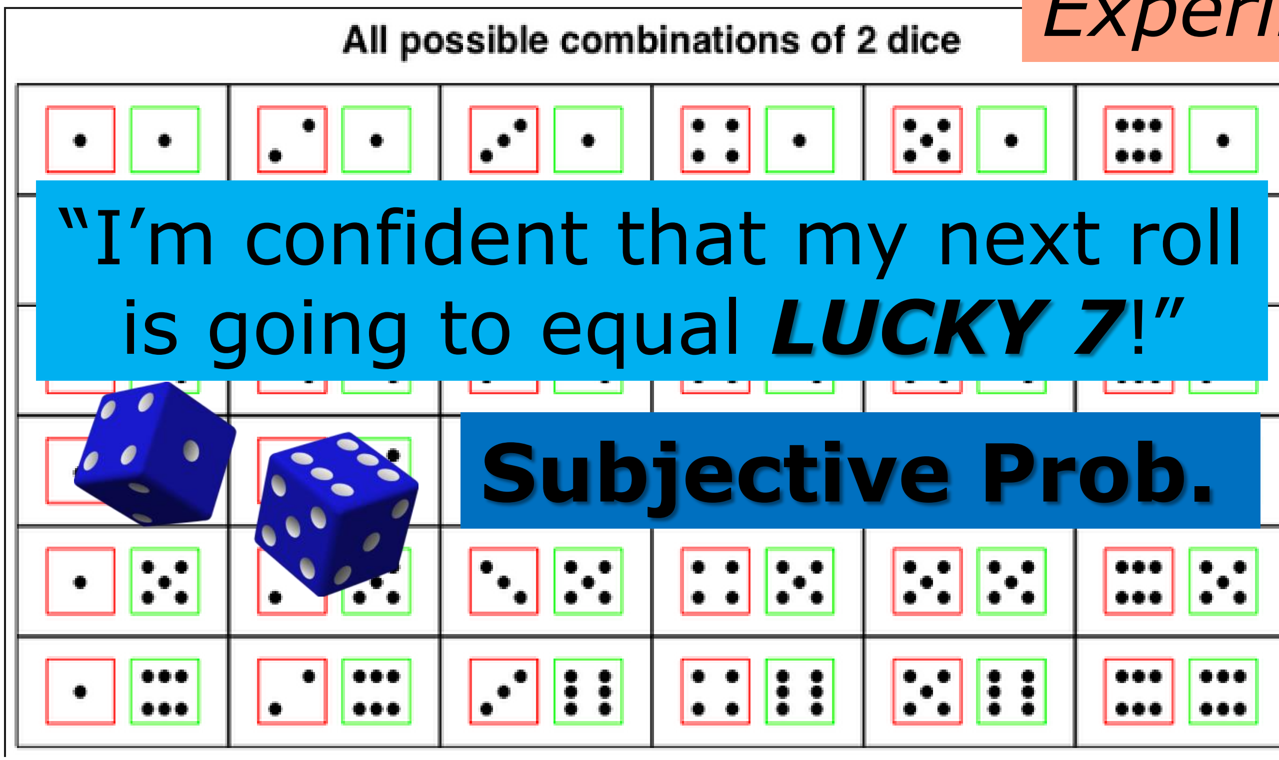
# Subjective Probability

- Going into last year's NCAA basketball tournament: What do you think the probability is for UK to win the Championship?
- **U of K Fan: "I think it is about 95%"**
- U of L Fan: "I guess it's a 60% to 70% chance that they'll win."
- Duke Fan: "I think it is about a 20% chance, especially if they have to play us!"

# Subjective Probability

- What do you think the probability is that the Chicago Cubs will win the World Series?
- General Baseball Fans: "I think there is about a 15% chance"
- **St. Louis Cardinal Fans: "There is probably a 10% chance"**
- **Cub Fans: "There almost no chance!** We're talking about the ***cursed*** **Cubbies!"**

# Sample Space of 2 Dice




“I’m confident that my next roll is going to equal ***LUCKY 7***!”

**Subjective Prob.**

*Experimental Prob.*

Rolled the dice 10 times:

1st: 2, 5
2nd: 3, 4
3rd: 1, 3
4th: 5, 5
5th: 6, 1
6th: 3, 6
7th: 5, 4
8th: 4, 4
9th: 1, 2
10th: 4, 3

P (sum 7) $= \frac{4}{10}$

P (sum of 7) $= \frac{6}{36} = \frac{\mathbf{1}}{\mathbf{6}}$

Theoretical Prob.

# Statistics & Probability App:

## Learning Statistics & Probability

Link: http://go.golearningbus.com/



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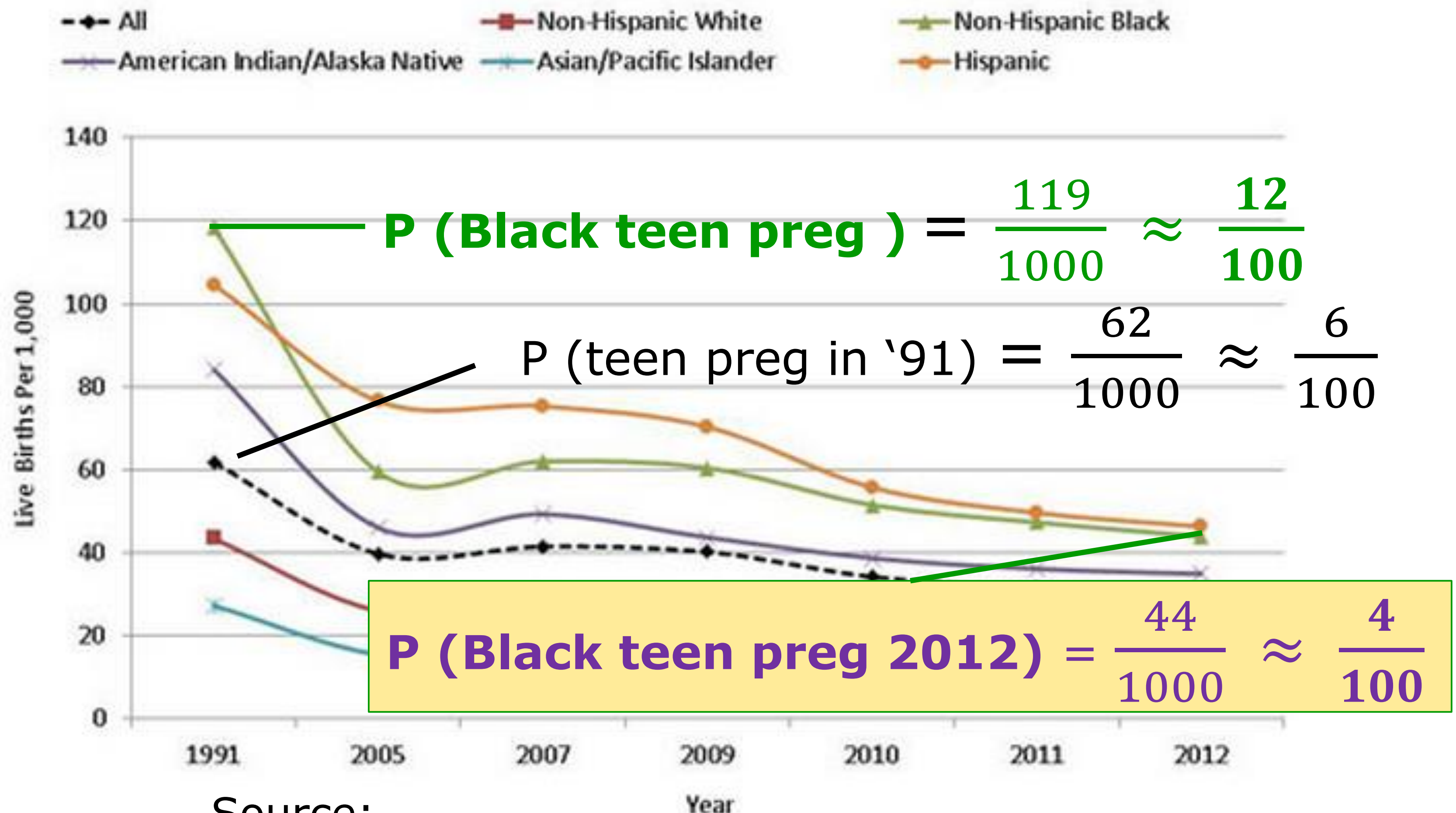
# Teenage Pregnancy Statistics (1991 -2012)




**P (Black teen preg )** $= \frac{119}{1000} \approx \frac{12}{100}$

P (teen preg in '91) $= \frac{62}{1000} \approx \frac{6}{100}$

**P (Black teen preg 2012)** $= \frac{44}{1000} \approx \frac{4}{100}$

Source: http://www.cdc.gov/teenpregnancy/about/birth-rates-chart-2000-2011-text.htm

# Teenage Pregnancy for Girls in Foster-Care

Source: Time Magazine (2009) ***Teen Pregnancy: An Epidemic in Foster Care***
By Amy Sullivan

- A study at the University of Chicago found that nearly half of girls who had spent time in the foster-care system had been pregnant at least once by the time they were 19 years old.

$$P\,(foster\ teen\ preg.) = \frac{500}{1000} = \frac{1}{2}$$

- Even more troubling— close to one-quarter had experienced multiple pregnancies in their teens.

$$P\,(foster\ teen\ \ mult.\,preg.) = \frac{240}{1000} \approx \frac{1}{4}$$

# College Graduation Rates

## Source: National Center for Educational Statistics

- **Question:**
  What are the graduation rates for students obtaining a bachelor's degree?
- **Response:**
  The 2012 graduation rate for first-time, full-time undergraduate students who began their pursuit of a bachelor's degree (B.A.) at a 4-year degree-granting institution in fall 2006 was 59 percent.
- $P\,(B.A.\,|\,first\ time, full)$

# Conditional Probability

- P(A | B) = the (conditional) probability that event A will occur given that event B has occurred already
- The usual notation for "event A occurs given that event B has occurred" is "A | B" (A given B). The symbol | is a vertical line and does not imply division. P(A | B) denotes the probability that event A will occur given that event B has occurred already.

# College Graduation Rates

## Source: National Center for Educational Statistics

- Gents: $P\,(B.A.|male) = \frac{56}{100}$
- Ladies: $P\,(B.A.|female) = \frac{61}{100}$
- Private: $P\,(B.A.|nonprofit) = \frac{66}{100}$
- Public: $P\,(B.A.|nonprofit) = \frac{57}{100}$
- Private $P\,(B.A.|\text{for−profit}) = \mathbf{\frac{32}{100}}$

# Compound Events

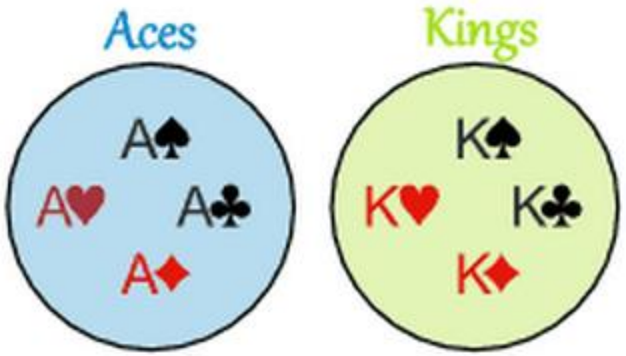
- The UNION (⋃) or INTERSECTION (⋂) of two events is called a **compound event**
- If P(A) = probability that event A occurs
- If P(B) = probability that event B occurs
- The **U**NION (⋃) of two event is the same as finding P (A *or* B);
- The INTERSECTION (⋂) of two event is the same as finding P (A *and* B);

# Compound Events

- The **addition rule** is a result used to determine the probability that event A or event B occurs or both occur; UNION (∪):
- $P(A \cup B) = P(A) + P(B) - P(A\ and\ B)$
- If the events do not share any outcomes in common (*mutually exclusive*), then the P(A or B) is simply $P(A \cup B) = P(A) + P(B)$

# Vocabulary for Compound Events

- Two events are **mutually exclusive** (or *disjoint*) if it is impossible for them to occur together.
- Example: Drawing one card from a deck that is both an Ace and a King
- Notation: P (A *and* B) = $P(A \cap B)$
- P (A *and* B) = 0



# Compound Event Example

- Suppose we wish to find the probability of drawing either a king or a spade in a single draw from a pack of 52 cards:
- We define the events:
- Event A = draw a king; and
  Event B = draw a spade
- so P(King or a Spade) can be written as $P(A \cup B)$

# Compound Events (cont.)

- $\mathrm{P}(\mathrm{King} \cup Spade) = P(A) + P(B) - P(A\ and\ B)$
- or $\mathrm{P}(\mathrm{A} \cup B) = P(A) + P(B) - P(A \cap B)$
- Since there are 4 kings in the pack and 13 spades, but 1 card is both a king and a spade, we have:
- $\mathrm{P}(\mathrm{King} \cup Spade) = P(A) + P(B) - P(A\ and\ B) =$ 4/52 + 13/52 - 1/52 = 16/52
- So, the probability of drawing either a king or a spade is 16/52 (= 4/13).

# Compound Events

- The multiplication rule is a result used to determine the probability that two events, A and B, both occur; INTERSECTION (∩)
- Notation: P (A *and* B) = $\mathrm{P}(A \cap B)$
- For **independent events**, that is events which have no influence on one another, the rule is P (A *and* B) = P (A)$\cdot \mathrm{P}(B)$

# Compound Events

- Given a six-sided die and a fair coin, what is the probability of rolling a 5 or 6 and getting tails?
- We define the events:
- Event A = rolling a 5 or 6; and
  Event B = coin lands on tails
- P (A *and* B) = P (A)$\cdot \mathrm{P}(B)$

$$\mathrm{P}(A \cap B) = \frac{2}{6} \cdot \frac{1}{2} = \frac{1}{6}$$

# Compound Events

- Two events *A* and *B* are called **independent events** if knowledge about the occurrence of one of them has no effect on the probability of the other one, that is, if
- $\boldsymbol{P(B \mid A) = P(B)}$, or equivalently
- $\boldsymbol{P(A \mid B) = P(A)}$.

# Independent events

- We define the events:
- Event A = rolling a 5 or 6; and
  Event B = coin lands on tails
- ***P*(*B* | *A*) = *P*(*B*):**
- $P\ (tails \mid roll\ 5\ or\ 6) = \frac{1}{2}$
- ***P*(*A* | *B*) = *P*(*A*):**
- $P\ (roll\ 5\ or\ 6 \mid tails) = \frac{2}{6} = \frac{1}{3}$

# Conditional Probability

- Sometimes the probability of an event must be computed using the knowledge that some other event has happened (or is happening, or will happen – the timing is not important). This type of probability is called ***conditional probability***.

# Conditional Probability

- The probability of event *B*, computed on the assumption that event *A* has happened, is called the **conditional probability of *B*, given *A*,** and is denoted ***P*(*B* | *A*).**
- What is the probability of drawing an Ace from a deck of 52 cards?
- What is the probability of drawing an Ace from a deck, given that you already drew an Ace?

# Conditional Probability

- The **conditional probability of *B*, given *A*,** and is given by

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A \text{ and } B)}{P(A)}.$$

P (Draw an Ace , given you drew an Ace)

$$P\,(B \mid A) = \qquad \frac{\mathrm{P}(A \cap B)}{P(A)} = \frac{\frac{4}{52} \cdot \frac{3}{51}}{\frac{4}{52}} = \frac{1}{17}$$

# Dependent events

- Two events are called **dependent events** if the occurrence of one affects the occurrence of the other.
- P (A *and* B) = P (A)$\cdot P(B \mid A)$ *or*
- P (A ∩ $B$) $= \frac{4}{52} \cdot \frac{1}{17} = \mathbf{\frac{1}{221}}$

# Example

From the sample space
$S = \{2, 3, 4, 5, 6, 7, 8, 9\}$, a single number is to be selected randomly.
Given the events
*A:* selected number is odd, and
*B:* selected number is a multiple of 3.

find each probability.

a) $P(B)$
b) $P(A \text{ and } B)$
c) $P(B \mid A)$

# Example Solutions

- *A:* selected number is odd, and
- *B:* selected number is a multiple of 3.

a) *B* = {3, 6, 9}, so *P*(*B*) = 3/8

b) *P*(*A* and *B*) = {3, 5, 7, 9}    {3, 6, 9} = {3, 9}, so

*P*(*A* and *B*) = 2/8 = 1/4

c) The given condition *A* reduces the sample space to {3, 5, 7, 9}, so *P*(*B* | *A*) = 2/4 = 1/2

# Odds

# Odds

- Another way to describe the chance of an event occurring is with **odds**. The odds in **favor** of an event is the ratio that compares the number of ways the event **can** occur to the number of ways the event **cannot** occur.

- We can determine odds using the following ratios:

$$\text{Odds in Favor} = \frac{\text{number of successes}}{\text{number of failures}}$$

$$\text{Odds against} = \frac{\text{number of failures}}{\text{number of successes}}$$

# Example

- Suppose we play a game with 2 number cubes.
- If the sum of the numbers rolled is 6 or less – **you win!**
- If the sum of the numbers rolled is not 6 or less – **you lose**

**In this situation we can express odds as follows:**

$$\text{Odds in favor} = \frac{\text{numbers rolled is 6 or less}}{\text{numbers rolled is not 6 or less}} \quad \frac{15}{21} = \frac{5}{7}$$

$$\text{Odds against} = \frac{\text{numbers rolled is not 6 or less}}{\text{numbers rolled is 6 or less}} \quad \frac{21}{15} = \frac{7}{5}$$

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# Example

- A bag contains 5 yellow marbles, **3 white** marbles, and 1 black marble. What are the odds drawing a **white** marble from the bag?

Odds in favor = $\dfrac{\text{number of white marbles}}{\text{number of non-white marbles}}$ $\dfrac{3}{6}$

Odds against = $\dfrac{\text{number of non-white marbles}}{\text{number of white marbles}}$ $\dfrac{6}{3}$

Therefore, **the odds for are 1:2**

and **the odds against are 2:1**

# Comments

- On the next  couple of slides are some practice problems…The answers are on the last slide…
- Do the practice and then check your answers…If you do not get the same answer you must question what you did…go back and problem solve to find the error…
- If you cannot find the error bring your work to me and I will help…

# Your Turn - Probability

- Find the probability of randomly choosing a **specific** marble from the given bag of red and white marbles.

| | | | |
|---|---|---|---|
| 1. | Number of **red marbles** | 16 | |
| | Total number of marbles | 64 | P(red) |
| 2. | Number of **red marbles** | 8 | |
| | Total number of marbles | 40 | P(white) |
| 3. | Number of white marbles | 7 | |
| | Total number of marbles | 20 | P(white) |
| 4. | Number of white marbles | 24 | |
| | Total number of marbles | 32 | P(red) |

# Your Turn - Odds

- Find the **favorable** odds of choosing the indicated letter from a bag that contains the letters in the name of the given state.

5. S; Mississippi $\frac{4}{7}$
6. N; Pennsylvania $\frac{3}{9}=\frac{1}{3}$
7. A; Nebraska $\frac{2}{6}=\frac{1}{3}$
8. G; Virginia $\frac{1}{7}$

# Your Turn

- You toss a six-sided number cube 20 times. For twelve of the tosses the number tossed was 3 or more.

9. What is the experimental probability that the number tossed was 3 or more?

10. What are the favorable odds that the number tossed was 3 or more?

# Your Turn Solutions

1. ¼
2. 4/5
3. 13/20
4. ¼
5. 4/7
6. 3/9 or 1/3
7. 2/6 or 1/3
8. 1/7
9. 3/5
10. 3/2

# Summary

- A key tool in making learning effective is being able to summarize what you learned in a lesson in your own words…

- In this lesson we talked about **probability and odds**… Therefore, in your own words summarize this lesson…be sure to include key concepts that the lesson covered as well as any points that are still not clear to you…