Probability Unit: What are the Chances?

Chance Experiments & Events
Probability Rules

Adapted from The Practice of Statistics, STARNES YATES, MOORE, 4th edition – For AP*

Concepts for Probability:
What Are the Chances? *(Dec. 1, 2022)*

- Review: Chance Experiments & Events
- **TODAY: Probability Rules**
- **Odds vs. Evens – Partner Activity**
- Probability QUIZ Today!

Warm-Up Dec 1, 2022: Consider flipping a coin 3 times

1) Ava tosses a fair coin three times.

a) Draw a tree diagram to show all the possible outcomes.
b) Find the probability of getting:
   (i) Three tails.
   (ii) Exactly two heads.
   (iii) At least two tails.

2) What are the *addition* & **multiplication rules** for probability?

Review of Probability Rules

Learning Objectives

After this Unit, you should be able to…

- ✓ DESCRIBE chance behavior with a probability model
- ✓ DEFINE and APPLY basic rules of probability
- ✓ DETERMINE probabilities from two-way tables
- ✓ CONSTRUCT Venn diagrams and DETERMINE probabilities

Addition Rule: to find $P(\mathrm{A} \cup \mathrm{B})$

Multiplication Rule: *to find* $P(A \cap B)$

- Addition Rule : The probability of the Union of two events is found by adding their separate probabilities, and then subtracting their intersection:

$$P(\mathrm{A} \cup \mathrm{B}) = P(A) + P(B) - P(A \cap B)$$

- Multiplication Rule: The probability of the i∩tersection of two events is found by multiplying the probability of one event times the conditional probability of the other:

$$P(\mathrm{A} \cap \mathrm{B}) = \mathrm{P}(\mathrm{A}) \cdot P(B|A) \text{ or } \mathrm{P}(B) \cdot P(A|B)$$

- If events are independent:

$$P(\mathrm{A} \cap \mathrm{B}) = \mathrm{P}(\mathrm{A}) \cdot P(B)$$

Probability Models

In Section 5.1, we used simulation to imitate chance behavior. Fortunately, we don't have to always rely on simulations to determine the probability of a particular outcome.

Descriptions of chance behavior contain two parts:

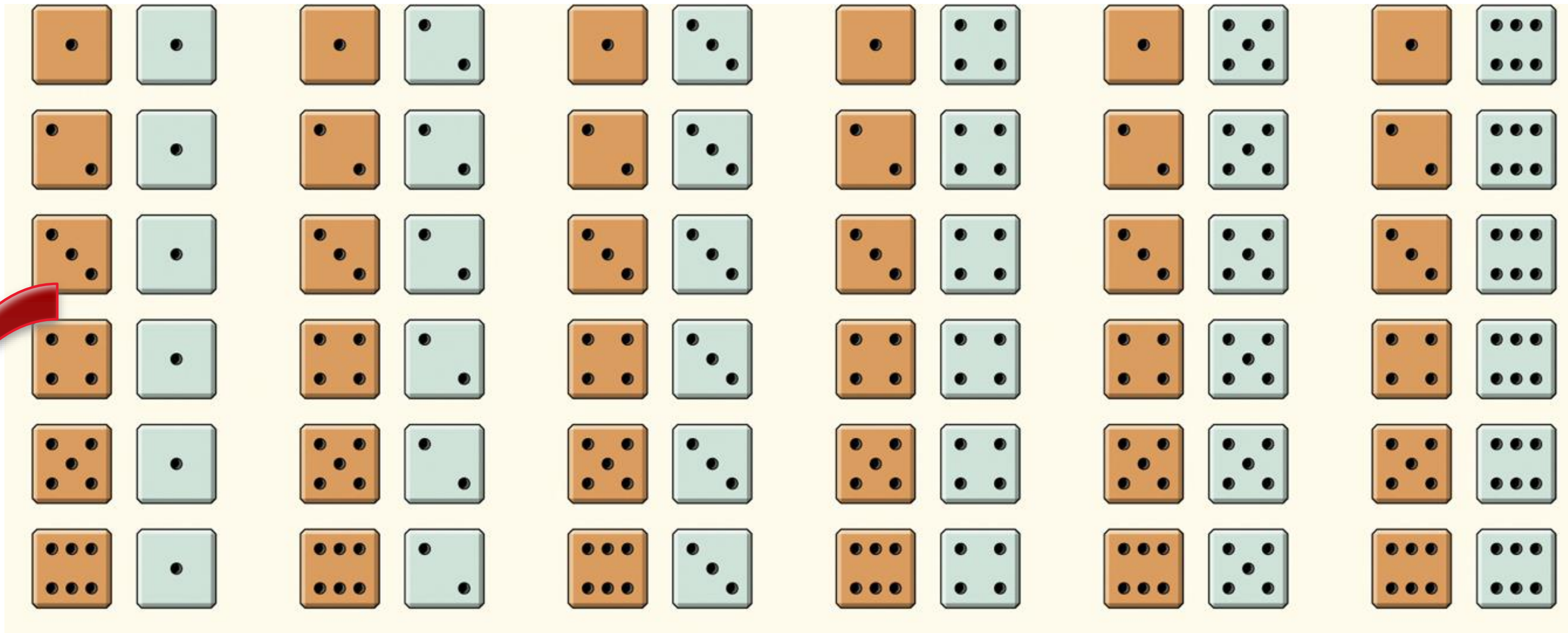
Definition:

The **sample space *S*** of a chance process is the set of all possible outcomes.

A **probability model** is a description of some chance process that consists of two parts: a sample space *S* and a probability for each outcome (probability distribution). The sum of the outcomes for the distribution must equal 1.

Example: Roll the Dice

Give a probability model for the chance process of rolling two fair, six-sided dice – one that's red and one that's green.



Sample Space 36 Outcomes

Since the dice are fair, each outcome is equally likely. Each outcome has probability 1/36.

Probability Models

Probability models allow us to find the probability of any collection of outcomes.

Definition:

An **event** is any collection of outcomes from some chance process. That is, an event is a subset of the sample space. Events are usually designated by capital letters, like *A, B, C,* and so on.

If *A* is any event, we write its probability as P(*A*).

In the dice-rolling example, suppose we define event *A* as “sum is 5.”

There are 4 outcomes that result in a sum of 5.
Since each outcome has probability 1/36, P(*A*) = 4/36.

Suppose event *B* is defined as “sum is not 5.” What is P(*B*)? *P*(*B*) = 1 – 4/36
= 32/36

Warm-Up:
Consider flipping a coin 3 times

Emily tosses a fair coin three times.

a) Draw a tree diagram to show all the possible outcomes.

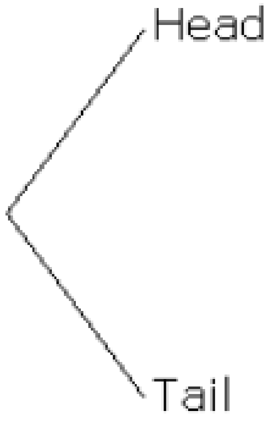
b) Find the probability of getting:
   (i) Three tails.
   (ii) Exactly two heads.
   (iii) At least two tails.

Consider flipping a coin 3 times

Solution:

a) A tree diagram of all possible outcomes.

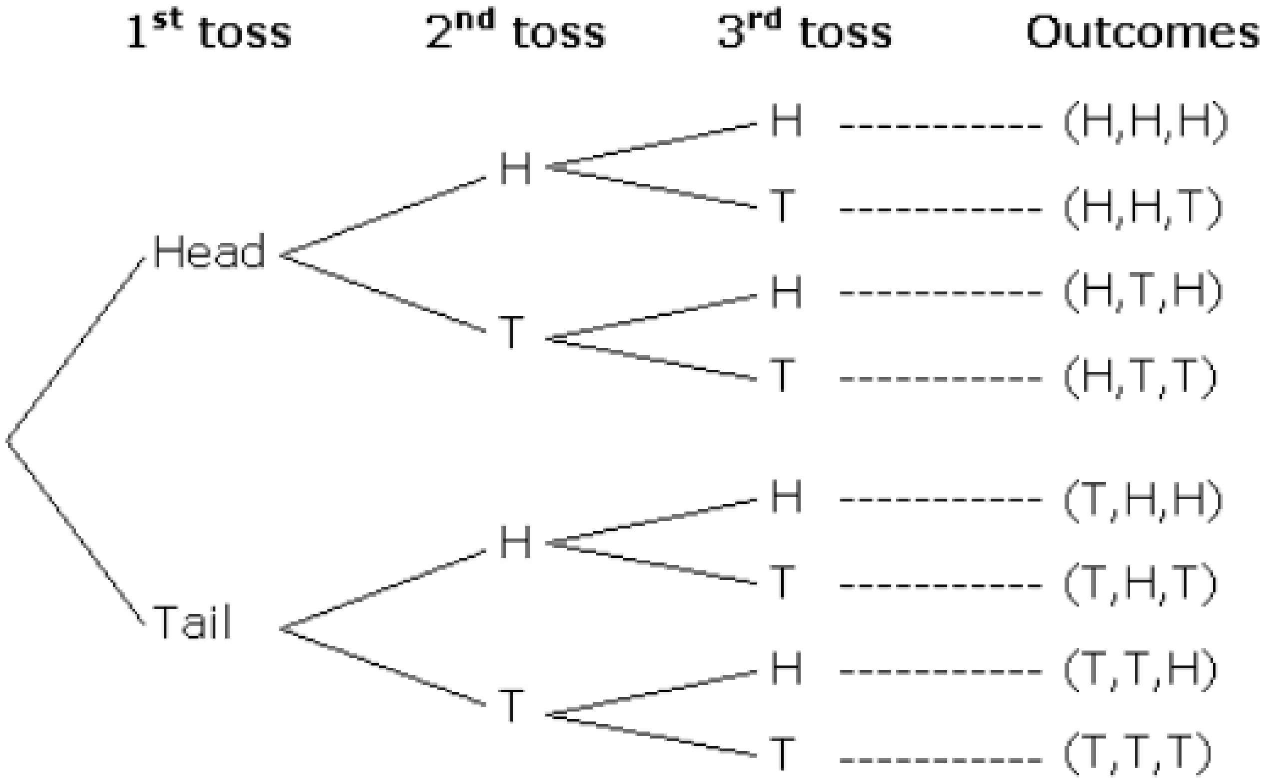
1st toss



Consider flipping a coin 3 times

Solution:

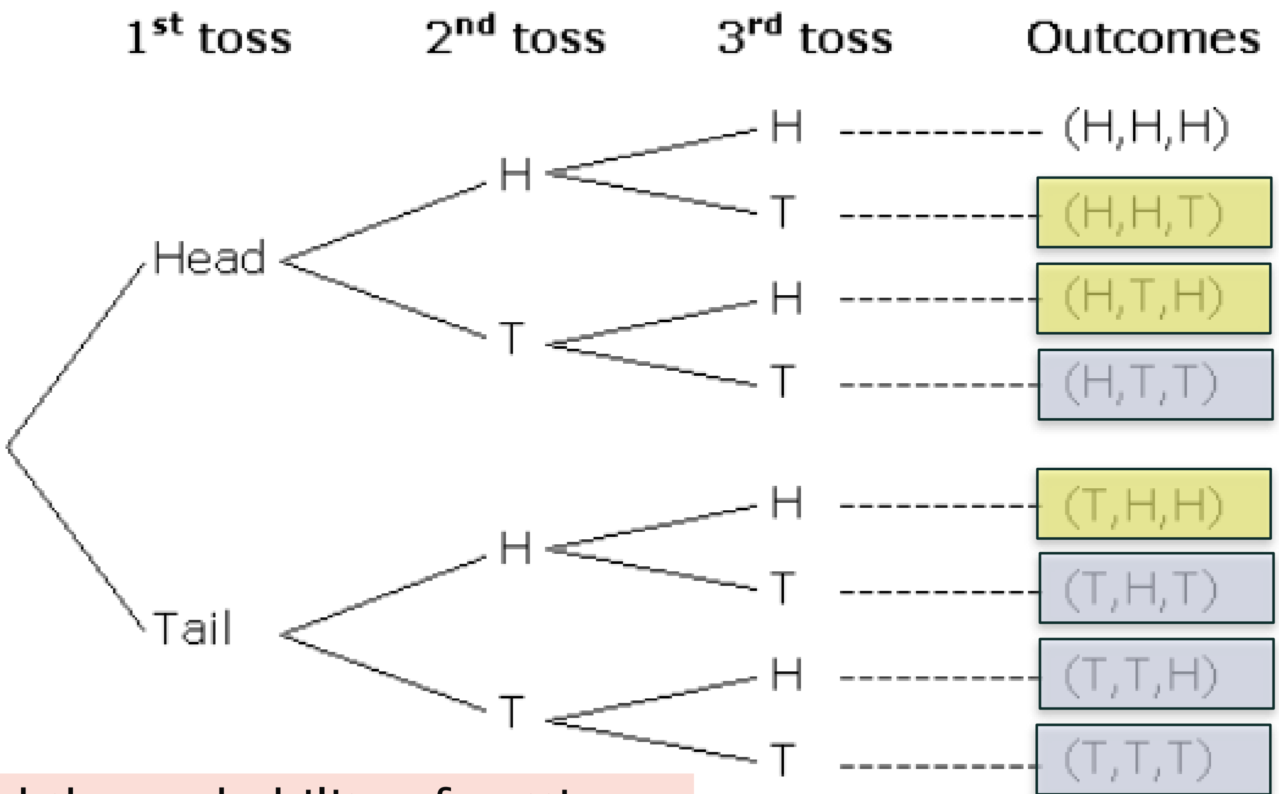
a) A tree diagram of all possible outcomes.



Consider flipping a coin 3 times

Solution:

a) A tree diagram of all possible outcomes.



Find the probability of getting:

(iii) At least two tails $= \frac{4}{8} = \mathbf{0.500}$

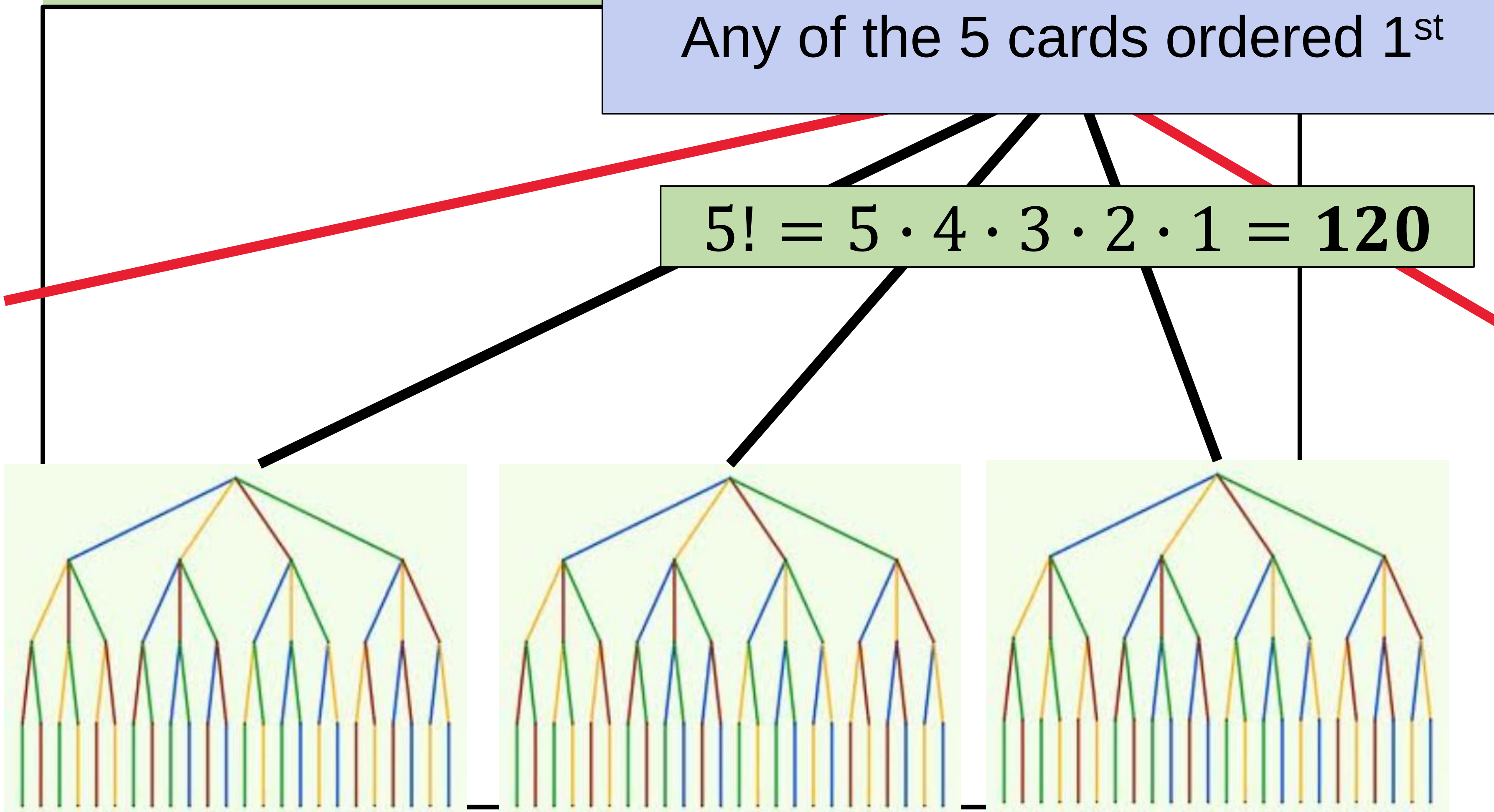
(two tails or *three tails)*

Tree diagrams: Graphical display to help organize all possible outcomes

How many ways can you order **5 cards**?

Any of the 5 cards ordered 1st

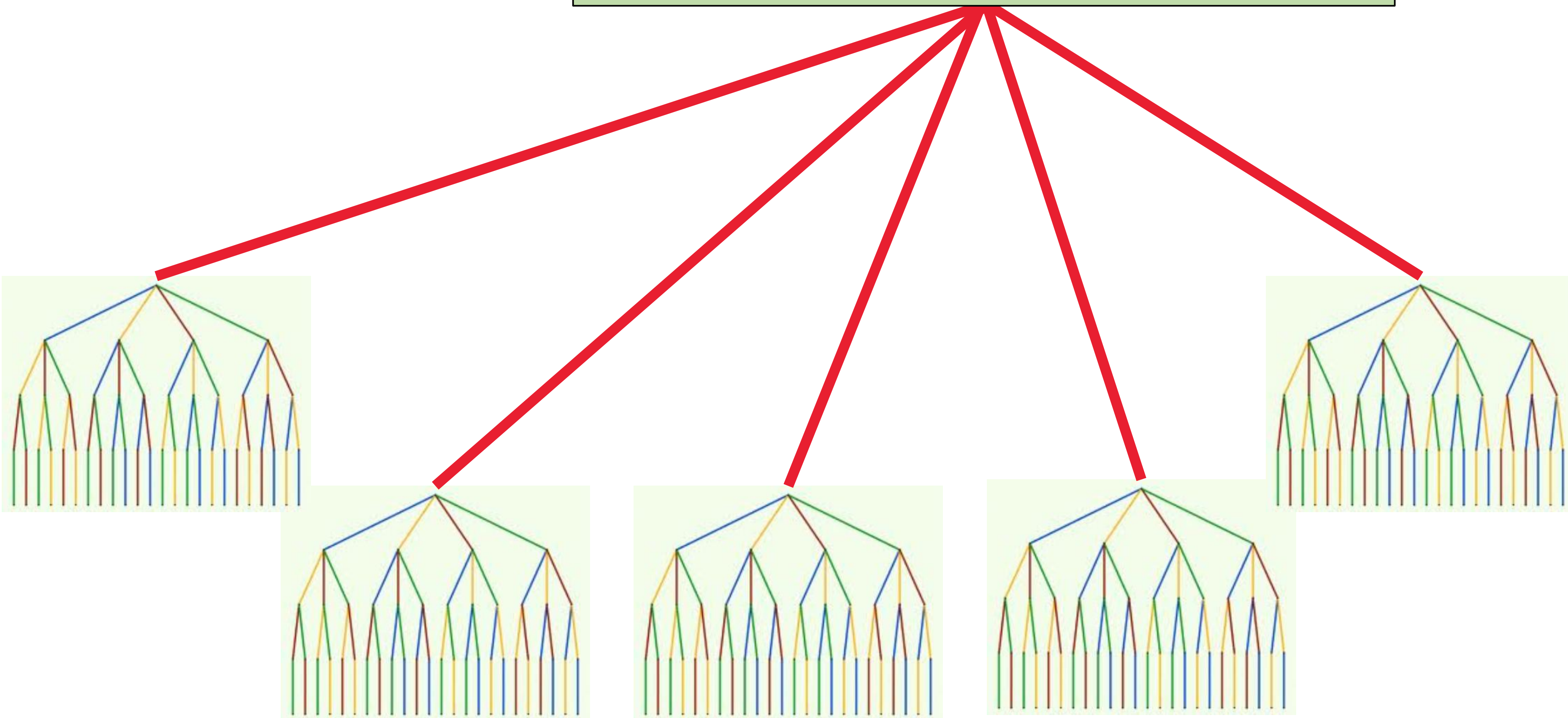
$$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = \mathbf{120}$$



Tree Diagram:

How many ways can you order **5 cards**?

$$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = \mathbf{120}$$



Partner WS: Odds or Evens

Get a pair of dice, then play the games and complete questions 1 to 4

Name: ______________________________ Block: _____ Date: ______

HW 19: Probability: Odds or evens, who will win?

We're going to play a game to answer this question. You and your partner must decide who will be "Odds" and who will be "Evens". Then you will roll two dice and **multiply** the numbers. If the product is odd, the odds person wins and vice versa for evens. Play 20 times, keeping track of how many wins each person has.

1. How many times did the odds win? __________ $\frac{x}{20} =$

   Write this as a fraction out of 20 and turn it to a percentage.

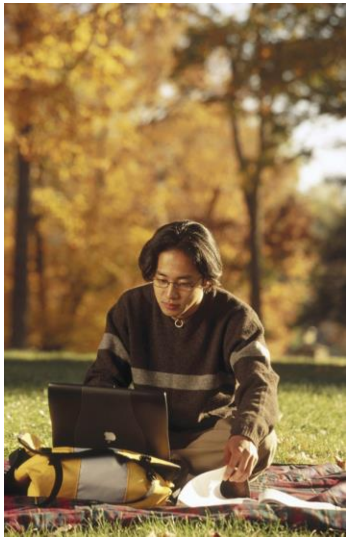
Maybe the odds just had a run of bad luck. Let's see how the rest of the class did with odds. Write the number of odds wins for your group in the table on the board.

2. Find the total percent of rolls that were odd products for the whole class. _______

Example: Distance Learning

Distance-learning courses are rapidly gaining popularity among college students. Randomly select an undergraduate student who is taking distance-learning courses for credit and record the student's age. Here is the probability model:

Age group (yr):	18 to 23	24 to 29	30 to 39	40 or over
Probability:	0.57	0.17	0.14	0.12



(a) Show that this is a legitimate probability model.

Each probability is between 0 and 1 and
0.57 + 0.17 + 0.14 + 0.12 = 1

(b) Find the probability that the chosen student is not in the traditional college age group (18 to 23 years).

$$P(\text{not 18 to 23 years}) = 1 - P(\text{18 to 23 years}) = 1 - 0.57 = 0.43$$

Basic Rules of Probability

All probability models must obey the following rules:

- **The probability of any event is a number between 0 and 1.**
- **All possible outcomes together must have probabilities whose sum is 1.**
- **If all outcomes in the sample space are equally likely, the probability that event *A* occurs can be found using the formula**

$$P(A) = \frac{\text{number of outcomes corresponding to event } A}{\text{total number of outcomes in sample space}}$$

- **The probability that an event does not occur is 1 minus the probability that the event does occur.**
- **If two events have no outcomes in common, the probability that one or the other occurs is the sum of their individual probabilities.**

Definition:

Two events are **mutually exclusive (disjoint)** if they have no outcomes in common and so can never occur together.

Basic Rules of Probability

- For any event A, $0 \le P(A) \le 1$.
- If S is the sample space in a probability model,

$$P(S) = 1.$$

- In the case of equally likely outcomes,

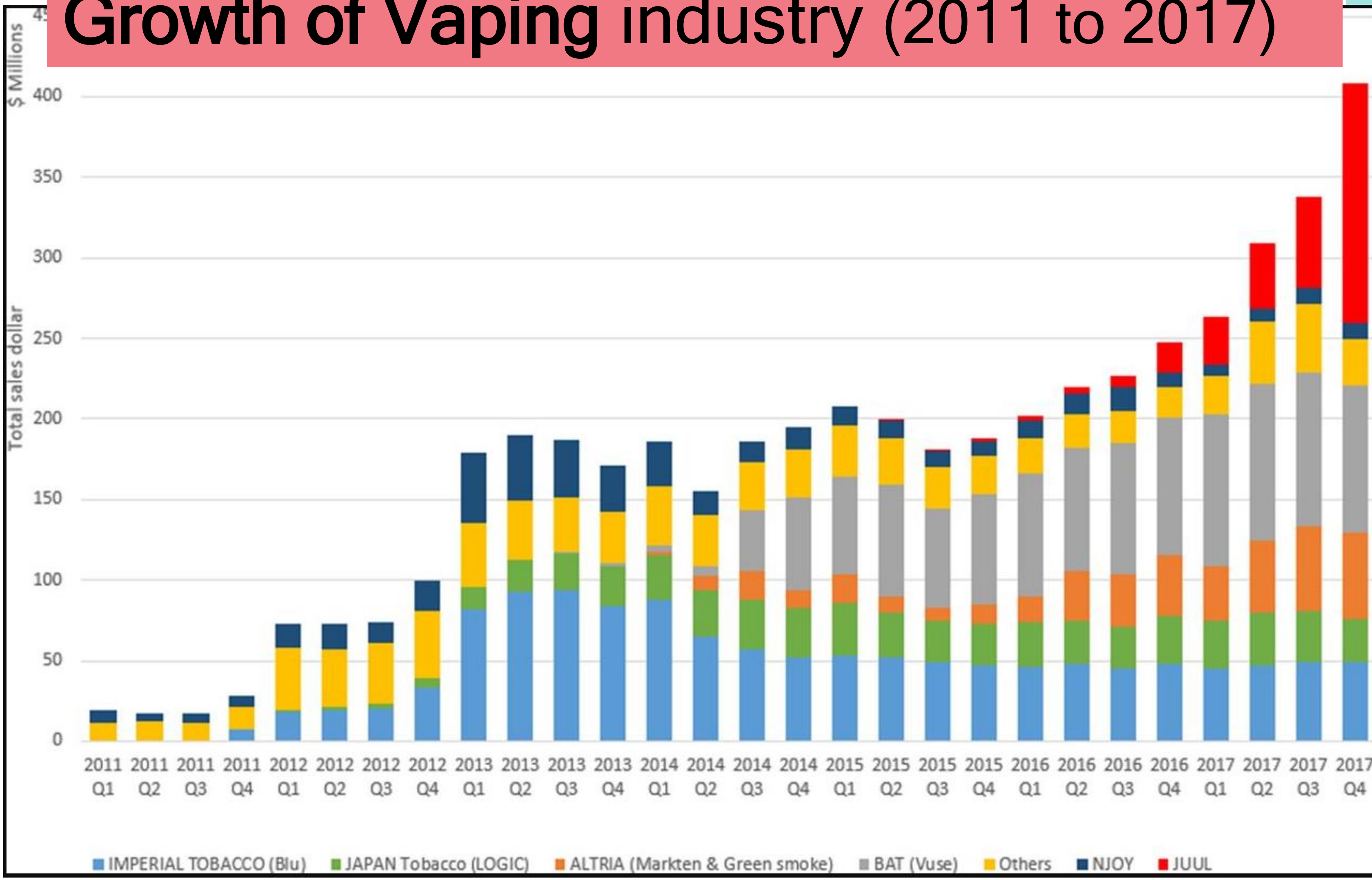
$$P(A) = \frac{\text{number of outcomes corresponding to event } A}{\text{total number of outcomes in sample space}}$$

- **Complement rule:** $P(A^C) = 1 - P(A)$
- **Addition rule for mutually exclusive events:** If A and B are mutually exclusive,

$$P(A \text{ or } B) = P(A) + P(B).$$

What does this image show?

Growth of Vaping industry (2011 to 2017)



Jidong Huang et al. Tob Control 2019;28:146-151

Two-Way Tables and Probability

When finding probabilities involving two events, a two-way table can display the sample space in a way that makes probability calculations easier.

Consider the example for students with pierced ears. Suppose we choose a student at random. Find the probability that a chosen student

	Pierced Ears?		
Gender	**Yes**	**No**	**Total**
Male	19	71	**90**
Female	84	4	**88**
Total	**103**	**75**	**178**

(a) has pierced ears.

(b) is a male with pierced ears.

(c) is a male or has pierced ears.

Define events *A*: is male and *B*: has pierced ears.

(a) Each student is equally likely to be chosen. 103 students have pierced ears. So, *P*(pierced ears) = *P*(*B*) = 103/178.

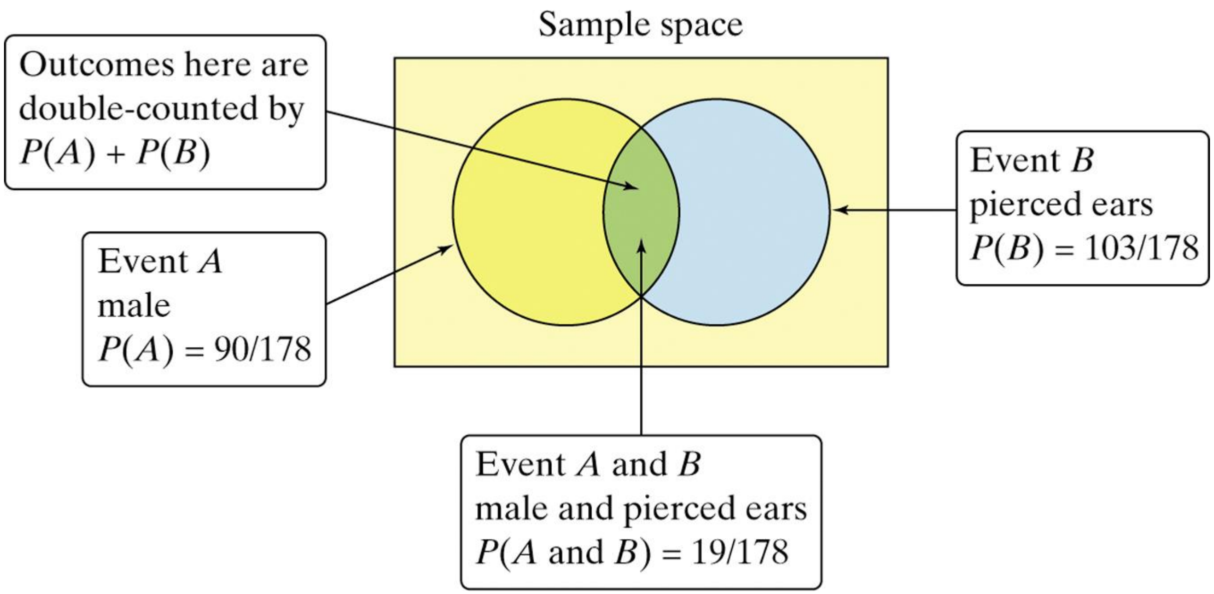
(b) We want to find *P*(male and pierced ears), that is, *P*(*A* and *B*). Look at the intersection of the "Male" row and "Yes" column. There are 19 males with pierced ears. So, *P*(*A* and *B*) = 19/178.

(c) We want to find *P*(male *or* pierced ears), that is, *P*(*A* or *B*). There are 90 males in the class and 103 individuals with pierced ears. However, 19 males have pierced ears – don't count them twice! *P*(*A* or *B*) = (19 + 71 + 84)/178. So, *P*(*A* or B) = 174/178

Two-Way Tables and Probability

Note, the previous example illustrates the fact that we can't use the basic addition rule for mutually exclusive events unless the events have no outcomes in common.

The **Venn diagram** below illustrates why.



General Addition Rule for Two Events

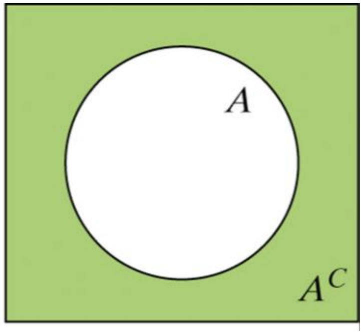
If *A* and *B* are any two events resulting from some chance process, then

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

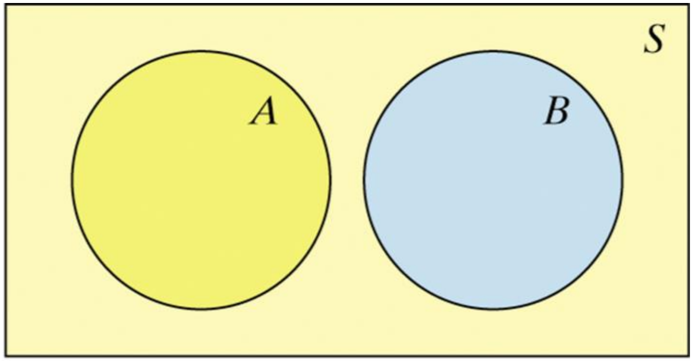
Venn Diagrams and Probability

Because Venn diagrams have uses in other branches of mathematics, some standard vocabulary and notation have been developed.

The complement A^C contains exactly the outcomes that are not in *A*.

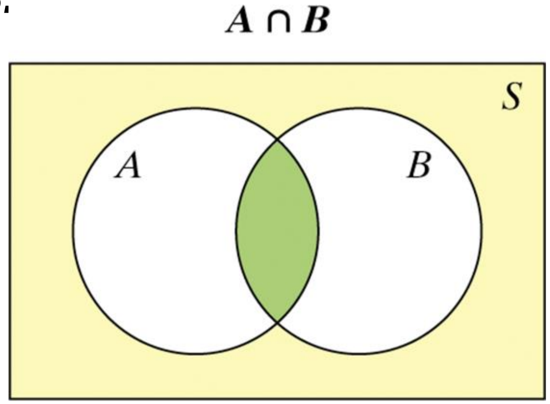


The events *A* and *B* are mutually exclusive (disjoint) because they do not overlap. That is, they have no outcomes in common.

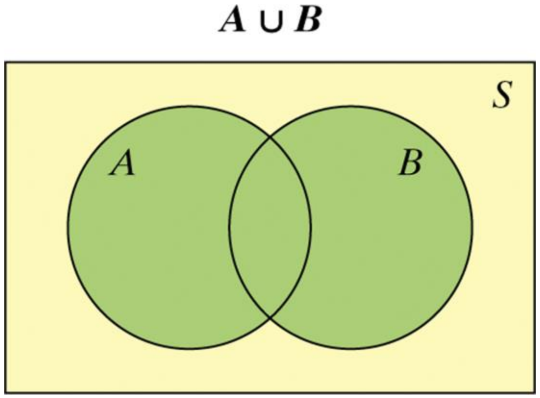


Venn Diagrams and Probability

The intersection of events *A* and *B* (*A* ∩ *B*) is the set of all outcomes in both events *A and B*.



The union of events *A* and *B* (*A* ∪ *B*) is the set of all outcomes in either event *A or B.*

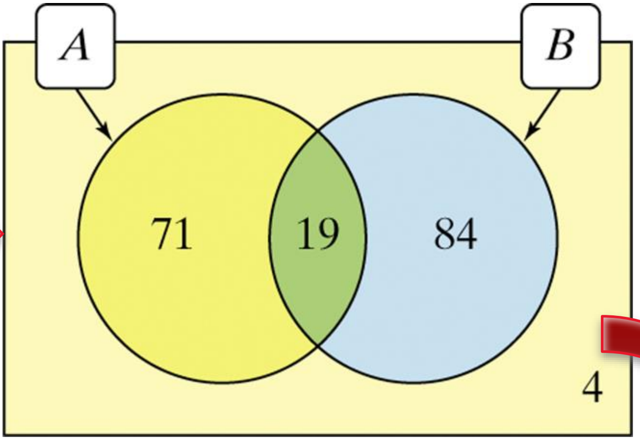


Hint: To keep the symbols straight, remember ∪ for union and ∩ for intersection.

Venn Diagrams and Probability

Recall the example on gender and pierced ears. We can use a Venn diagram to display the information and determine probabilities.

Gender	Pierced Ears? Yes	No	Total
Male	19	71	**90**
Female	84	4	**88**
Total	**103**	**75**	**178**



Define events *A*: is male and *B*: has pierced ears.

Region in Venn diagram	In words	In symbols	Count
In the intersection of two circles	Male and pierced ears	$A \cap B$	19
Inside circle *A*, outside circle *B*	Male and no pierced ears	$A \cap B^C$	71
Inside circle *B*, outside circle *A*	Female and pierced ears	$A^C \cap B$	84
Outside both circles	Female and no pierced ears	$A^C \cap B^C$	4

Conditional Probability – Car & Home owners

- Consider the population of U.S. adults. If we define events as

Event C: Car owner , and *Event H: Homeowner*

- *Then what is the probability that and adult owns a car or a home?* $P(C \cup H)$

- What is the *probability that and adult owns a car* ***and*** *a home?* $P(H \cap C)$

- *And what do the following conditional probabilities signify?*

$P(C \mid H)$ $P(H \mid C)$ $P(C \mid H^C)$

Conditional Probability – Car & Home owners

- Given Event C: Car owner , and *Event H: Homeowner, we need more information to solve any compound probabilities or conditional probabilities*

$P(C) = 0.85$ $P(H) = 0.64$

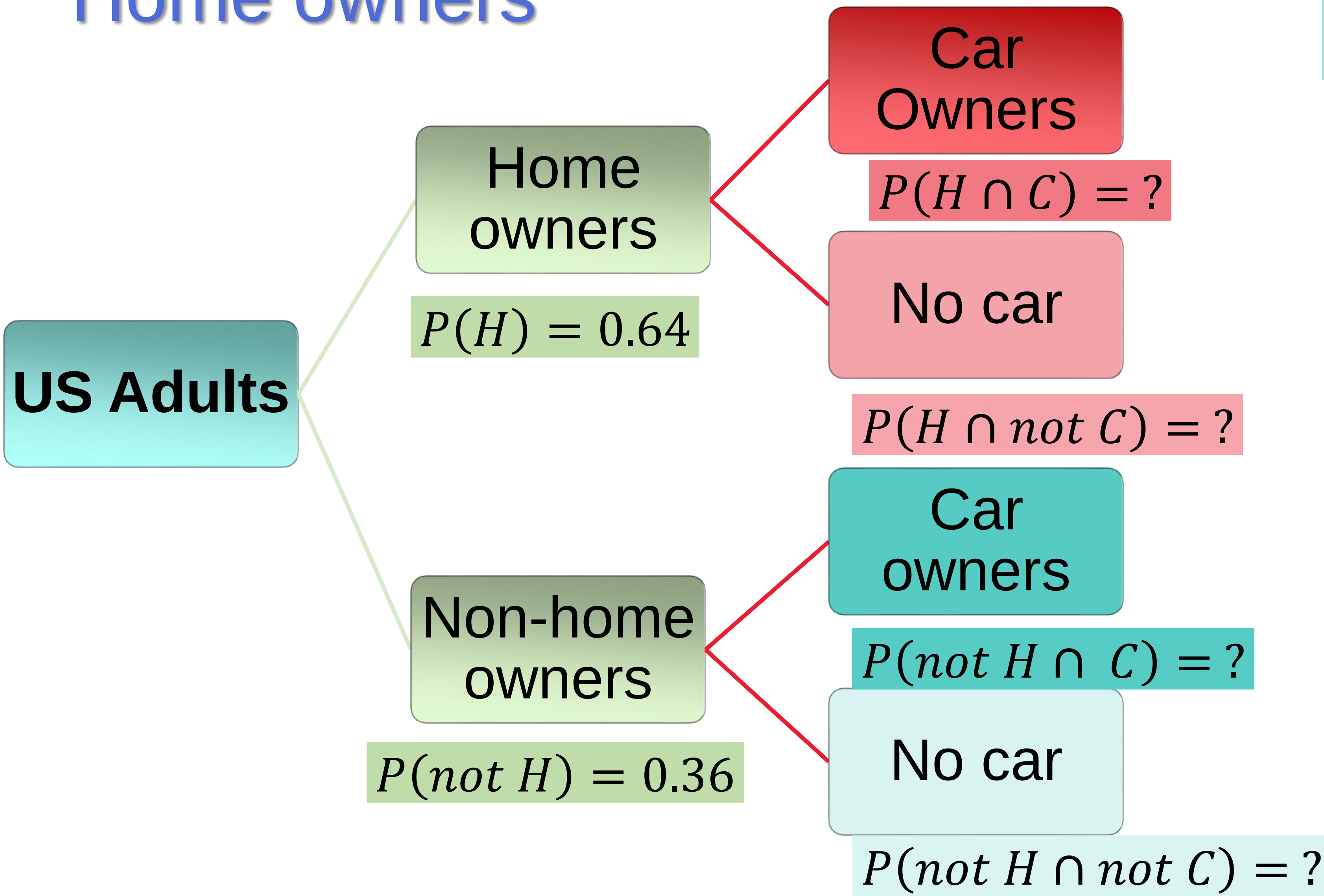
Can we now find the *probability that an adult owns either or both?*

$P(C \cup H)$ $P(H \cap C)$

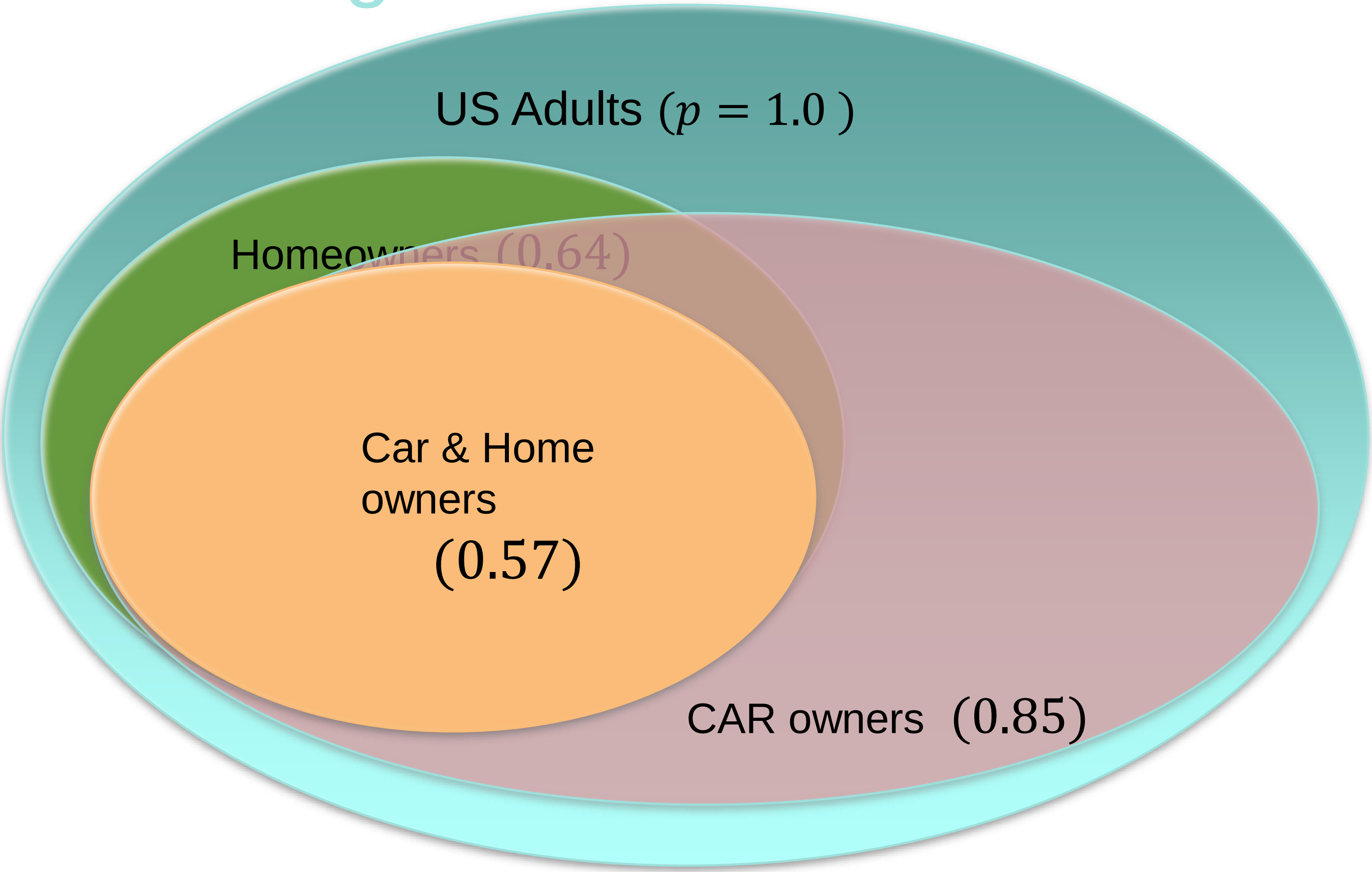
Are these events independent?

*Are they **mutually exclusive**?*

Conditional Probability – Car & Home owners



Conditional Probability
Venn Diagram



Symbolically $P(H \cap C)$

Conditional Probability

Population: U.S. Adults

Event C: Car owner

Event H: Home owner

P(Car owner)= *P*(**C**) = 0.85

P(Home owner) = *P*(**H**) = 0.64

P(Car owner *or* Homeowner)= $P(\text{C} \cup \text{H}) = P(C) + P(H) - P(H \cap C)$

$\text{P}(\text{C}|\text{H}) = \text{P}(\text{Car owner given that you own a home})$

$$\text{P}(\text{C}|\text{H}) = \frac{P(\text{C} \cap \text{H})}{P(\text{H})} = \frac{0.57}{0.64} = 0.89$$

$\text{P}(\text{H}|\text{C}) = P(\text{Home owner given that you own a car})$

+

Section 6.2
Probability Rules

Summary

In this section, we learned that…

- ✓ A **probability model** describes chance behavior by listing the possible outcomes in the **sample space *S*** and giving the probability that each outcome occurs.
- ✓ An **event** is a subset of the possible outcomes in a chance process.
- ✓ For any event A, $0 \le P(A) \le 1$
- ✓ $P(S) = 1$, where S = the sample space
- ✓ If all outcomes in S are equally likely, $P(A) = \frac{\text{number of outcomes corresponding to event } A}{\text{total number of outcomes in sample space}}$
- ✓ $P(A^C) = 1 - P(A)$, where A^C is the **complement** of event A; that is, the event that A does not happen.

Section 6.2 & 6.3
Probability Rules

Summary

In this section, we learned that…

- ✓ Events A and B are **mutually exclusive (disjoint)** if they have no outcomes in common. If A and B are disjoint, $P(A \text{ or } B) = P(A) + P(B)$.
- ✓ A **two-way table** or a **Venn diagram** can be used to display the sample space for a chance process.
- ✓ The **intersection ($A \cap B$)** of events A and B consists of outcomes in both A and B.
- ✓ The **union ($A \cup B$)** of events A and B consists of all outcomes in event A, event B, or both.
- ✓ The **general addition rule** can be used to find $P(A \text{ or } B)$:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Looking Ahead…

<u>In the next Section…</u>

We'll learn how to calculate conditional probabilities as well as probabilities of independent events.

We'll learn about

- ✓ **Conditional Probability**
- ✓ **Independence**
- ✓ **Tree diagrams and the general multiplication rule**
- ✓ **Special multiplication rule for independent events**
- ✓ **Calculating conditional probabilities**