Chapter 4: Numerical Methods for Distributions of Data

Interpreting Center & Variability in a Distribution

Adapted fromStatistics and Data Analysis, 5th edition - For AP*

PECK, OLSEN, & DEVORE

Warm-UP: Nov 3, 2021

- 1. What is meant by the statement: “Sara is in the 84th percentile of heights for girls of the same age”?
- 2. Define the standard deviation of a sample.
- 3. What are the three common measures of position for observations in a data set?
- 4. What is normal? What is a Normal distribution?

Warm-UP: Nov, 2020

- 1. “Sara is in the 84th percentile of heights” means that she is ***as tall or taller*** than 84 percent of the girls her same age.
- 2. Define the standard deviation of a sample:

$s_x = sample\ standard\ deviation$

A **statistic** that measures the typical distance from the mean for values (observations) in a distribution. It is calculated by finding the “average” of the squared distances, and then taking the square root

Warm-Up (cont.)

3. What are the three common measures of position for observations in a data set?

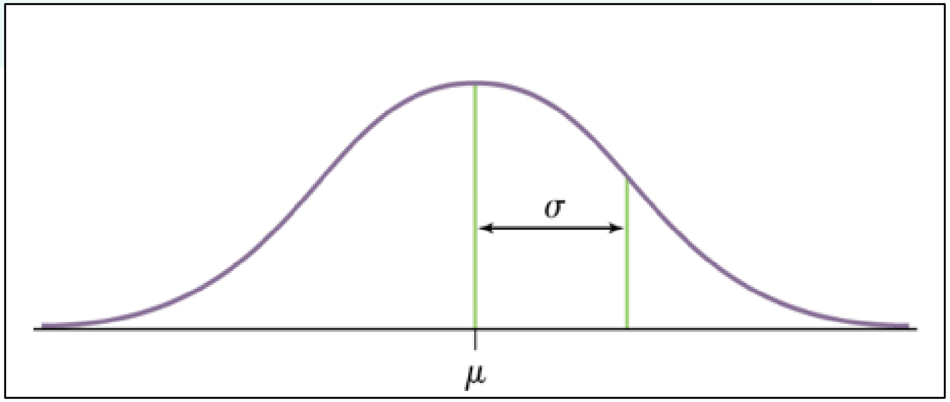
1. Percentiles
2. Quartiles
3. Standard scores (or *z-scores*)

Note: a z-score (or standard score) is a measure of position for an observation within a data set that provides a "standardized" measure of *distance* and *direction* in relation to The mean of the data, in terms of standard deviation

Warm-UP: Nov, 2021

- 3. Normal is what you are accustomed to experiencing. Maybe eating eggs and bacon every morning is “normal” for you. Maybe you normally eat cereal with almond milk. Maybe normal breakfast is a bowl of rice and fried fish.

- A Normal distribution is the commonly referred parametric distribution in statistics. It is symmetric, bell-shaped, and has equivalent measures of center (mean = median= mode). Every Normal distribution is clearly defined by the value of it’s mean and its standard deviation.



Modeling Distributions of Data

- **Describing Location in a Distribution**
- Density Curves
- Normal Distributions
- The Empirical Rule
- Calculating z Scores

Section 4.4
Describing Location in a Distribution

Learning Objectives

After this section, you should be able to…

- ✓ MEASURE position using percentiles
- ✓ MEASURE position using *z*-scores
- ✓ TRANSFORM data (z-scores)
- ✓ DEFINE and DESCRIBE density curves

Measuring Position: Percentiles

- One way to describe the location of a value in a distribution is to tell what percent of observations are less than it.

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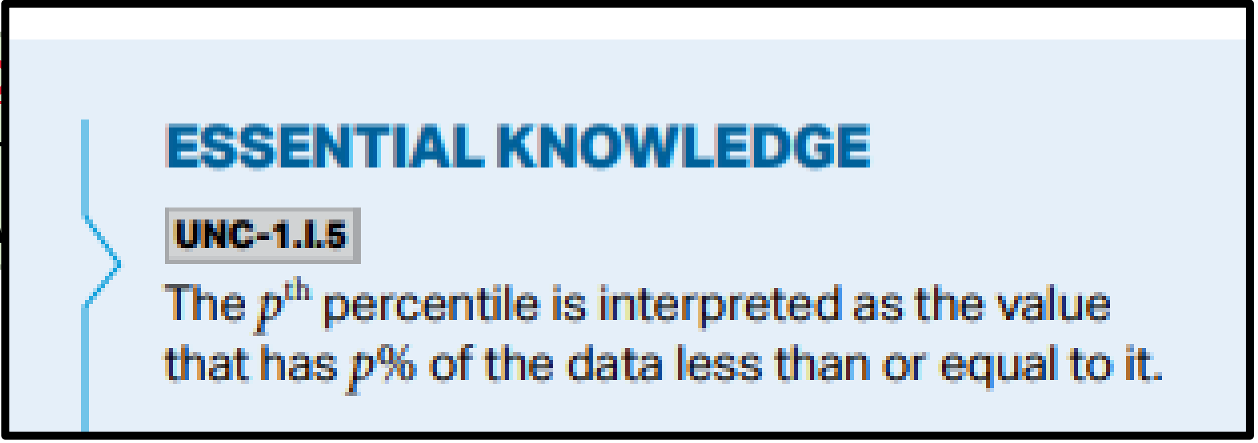


ESSENTIAL KNOWLEDGE

UNC-1.I.5

The p^{th} percentile is interpreted as the value that has p% of the data less than or equal to it.

Example

Jenny earned a score of 86 on her test. How did she perform relative to the rest of the class?

6 | 7
7 | 2334
7 | 5777899
8 | 00123334
8 | 569
9 | 03

Her score was greater than 21 of the 25 observations. Since 21 of the 25, or 84%, of the scores are below hers, Jenny is at the 84th percentile in the class's test score distribution.

Percentiles: 2 DEFINITIONS OF PERCENTILE

NOTE: There is no universally accepted, single definition of a percentile.

Definition 1 : Using the 65th percentile as an example, the 65th percentile can be defined *as the lowest score that is **greater than 65%** of the scores*.

Definition 2: The 65th percentile can also be defined as the smallest score **that is greater than or equal to 65%** of the scores.

"Unfortunately, these two definitions can lead to dramatically different results, especially when there is relatively little data. Moreover, neither of these definitions is explicit about how to handle rounding.

Measuring Position: *Z*-Scores

- A *z*-score tells us how many standard deviations from the mean an observation falls, and in what direction.

Definition:

If *x* is an observation from a distribution that has known mean and standard deviation, the **standardized value** of *x* is:

$$z = \frac{x - \text{mean}}{\text{standard deviation}}$$

A standardized value is often called a ***z*-score.**

Jenny earned a score of 86 on her test. The class mean is 80 and the standard deviation is 6.07. What is her standardized score?

$$z = \frac{x - \text{mean}}{\text{standard deviation}} = \frac{86 - 80}{6.07} = 0.99$$

Using *z*-scores for Comparison

We can use *z*-scores to compare the position of individuals in different distributions.

Example

Jenny earned a score of 86 on her statistics test. The class mean was 80 and the standard deviation was 6.07. She earned a score of 82 on her chemistry test. The chemistry scores had a fairly symmetric distribution with a mean 76 and standard deviation of 4. On which test did Jenny perform better ***relative to the rest of her class***?

$$z_{stats} = \frac{86-80}{6.07}$$

$$z_{stats} = 0.99$$

$$z_{chem} = \frac{82-76}{4}$$

$$z_{chem} = 1.5$$

Z-score WS practice: $z = \frac{obsev. - mean}{S.D.}$

1. A normal distribution of scores has a standard deviation of 10. Find the z-scores corresponding to each of the following values:

   a) A score that is 20 points above the mean.

   b) A score that is 10 points below the mean.

   c) A score that is 15 points above the mean

   d) A score that is 30 points below the mean.

Z-score WS practice: $z = \frac{obsev. - mean}{S.D.}$

The Welcher Adult Intelligence Test Scale is composed of a number of subtests. On one subtest, the raw scores have **a mean of 35** and a **standard deviation of 6.** Assuming these raw scores form a normal distribution:

a) What number represents the 65th percentile (what number separates the lower 65% of the distribution)?

Percentile	Quartile	z-score
65th	$>$ ☐ ***but*** $<$ ☐	**?**

Density Curves

- In Chapter 1, we developed a kit of graphical and numerical tools for describing distributions. Now, we'll add one more step to the strategy.

Exploring Quantitative Data

1. Always plot your data: make a graph.
2. Look for the overall pattern (shape, center, and spread) and for striking departures such as outliers.
3. Calculate a numerical summary to briefly describe center and spread.

4. Sometimes the overall pattern of a large number of observations is so regular that we can describe it by a smooth curve.

Density Curve

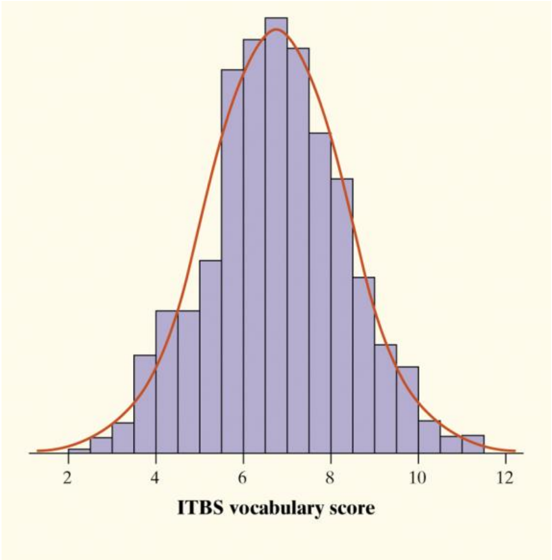
Definition:

A **density curve** is a curve that

- is always on or above the horizontal axis, and
- has area of ***exactly 1*** underneath it.

A density curve describes the overall pattern of a distribution. The area under the curve and above any interval of values on the horizontal axis is the proportion of all observations that fall in that interval.

The overall pattern of this histogram of the scores of all 947 seventh-grade students in Gary, Indiana, on the vocabulary part of the Iowa Test of Basic Skills (ITBS) can be described by a smooth curve drawn through the tops of the bars.



Describing Density Curves

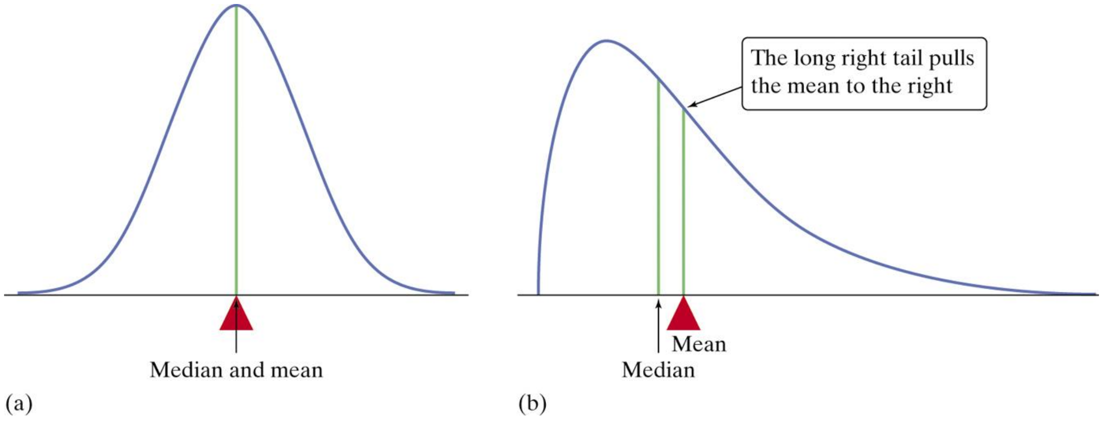
- Our measures of center and spread apply to density curves as well as to actual sets of observations.

Distinguishing the Median and Mean of a Density Curve

The **median** of a density curve is the equal-areas point, the point that divides the area under the curve in half.

The **mean** of a density curve is the balance point, at which the curve would balance if made of solid material.

The median and the mean are the same for a symmetric density curve. They both lie at the center of the curve. The mean of a skewed curve is pulled away from the median in the direction of the long tail.



Review of Position
Describing Location in a Distribution

Summary

In this section, we learned that…

- ✓ There are two ways of describing an individual's location within a distribution – the **percentile** and ***z*-score**..
- ✓ It is common to **transform data**, especially when changing units of measurement. Transforming data can affect the shape, center, and spread of a distribution.
- ✓ We can sometimes describe the overall pattern of a distribution by a **density curve** (an idealized description of a distribution that smooths out the irregularities in the actual data).

Looking Ahead…

In the next Section…

We'll learn about one particularly important class of density curves – the **Normal Distributions**

We'll learn

- ✓**The 68-95-99.7 Rule**
- ✓**The Standard Normal Distribution**
- ✓**Normal Distribution Calculations, and**
- ✓**Assessing Normality**

Normal Distributions (cont.)

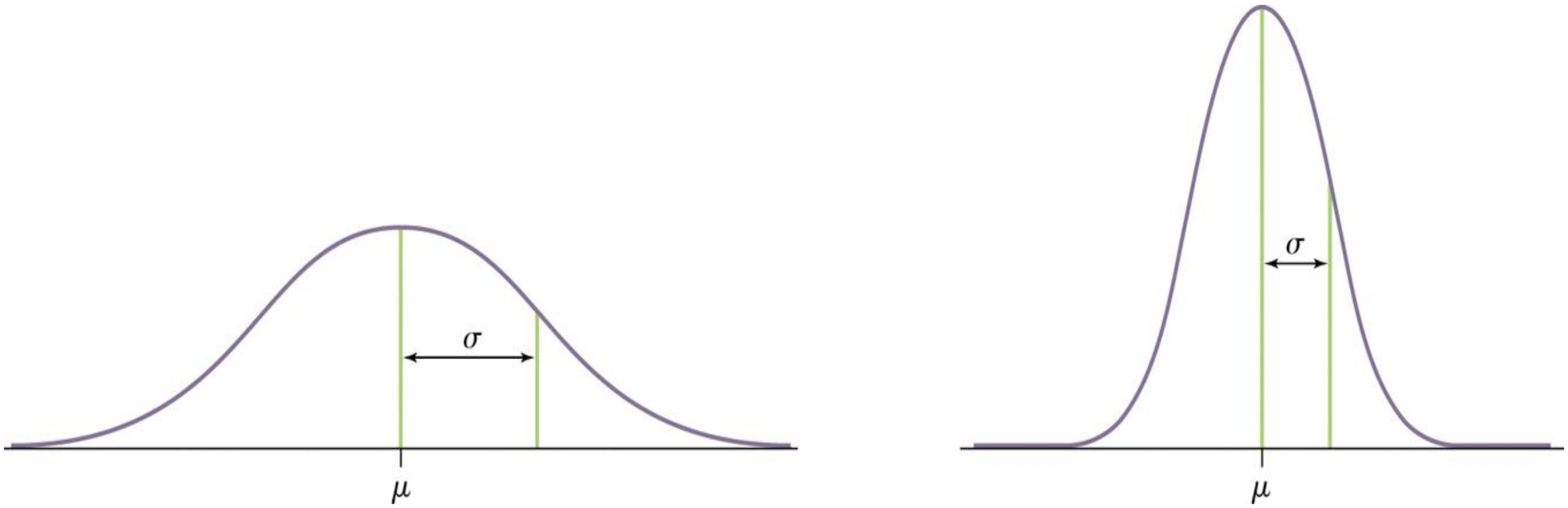
Learning Objectives

After this section, you should be able to…

- ✓ DESCRIBE and APPLY the Empirical Rule (68-95-99.7 Rule)
- ✓ DESCRIBE the standard Normal Distribution
- ✓ PERFORM Normal distribution calculations
- ✓ ASSESS Normality

Normal Distributions

- One particularly important class of density curves are the **Normal curves**, which describe **Normal distributions**.
  - All Normal curves are symmetric, single-peaked, and bell-shaped
  - Any Specific Normal curve is described by giving its mean μ *(“mu”)* and standard deviation σ (“sigma”).



Two Normal curves, showing the mean μ and standard deviation σ.

Normal Distributions

Definition:

A **Normal distribution** is described by a Normal density curve. Any particular Normal distribution is completely specified by two parameters: its **mean μ** *("mu")* and standard deviation σ ("sigma").

- The **mean** (μ) of a Normal distribution is the center of the symmetric **Normal curve**.
- The standard deviation (σ) is the distance from the center to the change-of-curvature points (*points of inflection*) on either side.
- We abbreviate the ***N***ormal distribution with mean and standard deviation as: $\boldsymbol{N}(\mu,\sigma)$.

Normal distributions are good descriptions for some distributions of *real data.*

Normal distributions are good approximations of the results of many kinds of *chance outcomes*.

Most of our *statistical inference* procedures are based on Normal distributions.

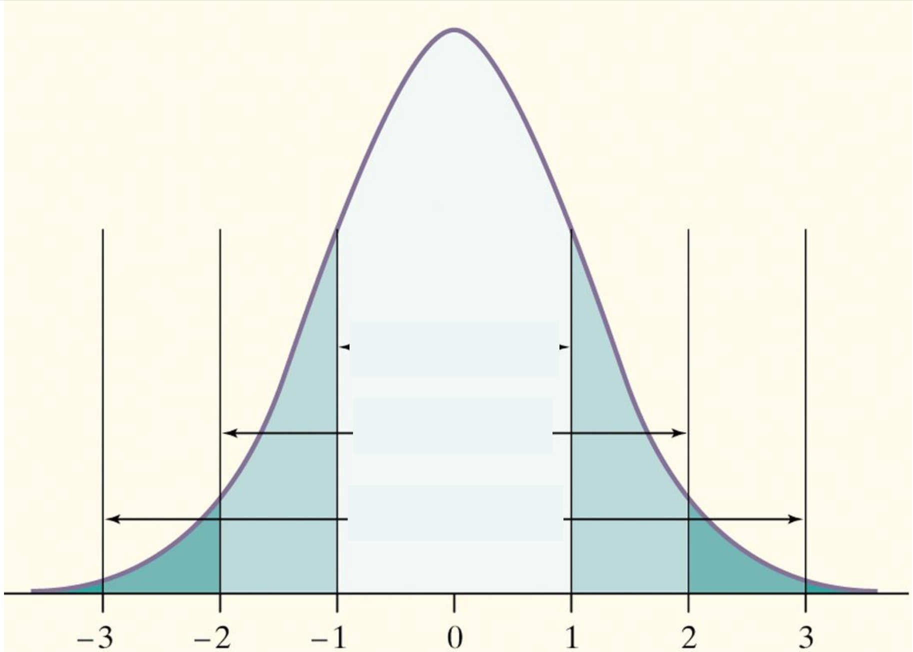
The Empirical Rule (68-95-99.7 Rule)

Although there are many Normal curves, they all have properties in common.

Definition: **The 68-95-99.7 Rule ("The Empirical Rule")**

In the Normal distribution with mean *μ* and standard deviation σ:

- Approximately **68%** of the observations fall within 1σ of *μ.*
- Approximately **95%** of the observations fall within 2σ of *μ.*
- Approximately **99.7%** of the observations fall within 3σ of *μ.*



- **The Standard Normal Distribution**
  - All Normal distributions are the same if we measure in units of size σ from the mean *μ* as center.

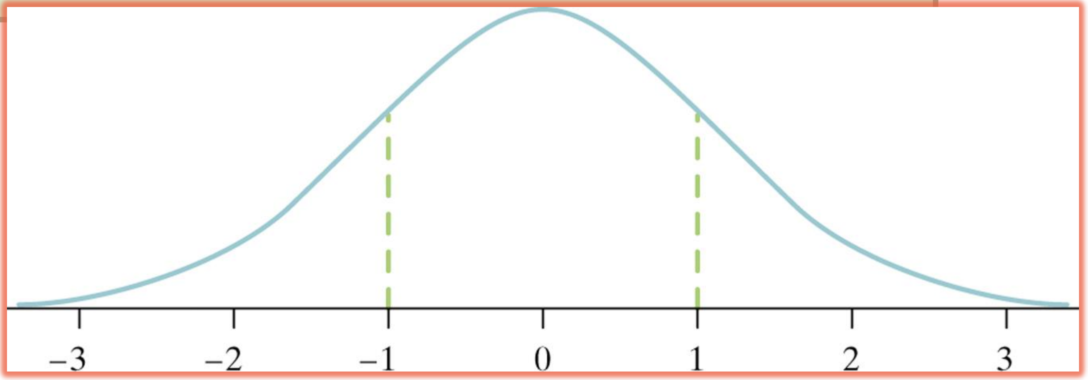
<u>**Definition:**</u>

The **standard Normal distribution** is the Normal distribution with mean 0 and standard deviation 1.

If a variable *x* has any Normal distribution *N*(*μ*,*σ*) with mean *μ* and standard deviation σ, then the standardized variable

$$z = \frac{x - \mu}{\sigma}$$

has the standard Normal distribution, *N*(0,1).



Normal Distribution Calculations

How to Solve Problems Involving Normal Distributions

STEP 4

State: Express the problem in terms of the observed variable *x*.

Plan: Draw a picture of the distribution and shade the area of interest under the curve.

Do: Perform calculations.

- ***Standardize*** *x* to restate the problem in terms of a standard Normal variable *z*.
- ***Use Standard Table*** and the fact that the total area under the curve is 1 to find the required area under the standard Normal curve.

Conclude: Write your conclusion in the context of the problem.

Example

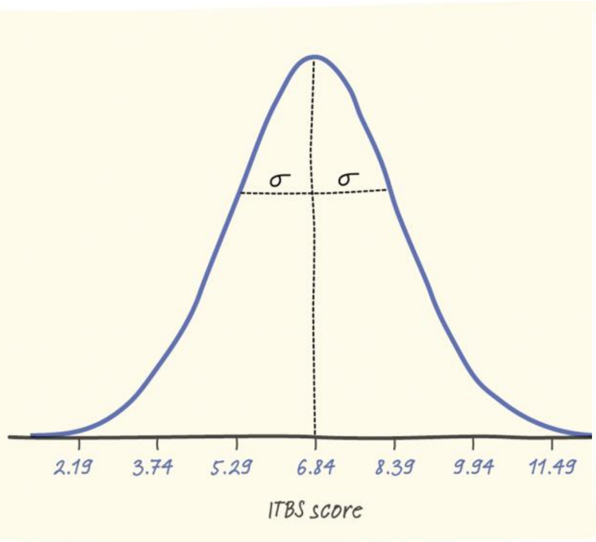
The distribution of Iowa Test of Basic Skills (ITBS) vocabulary scores for 7th grade students in Gary, Indiana, is close to Normal. Suppose the distribution is *N*(6.84, 1.55).

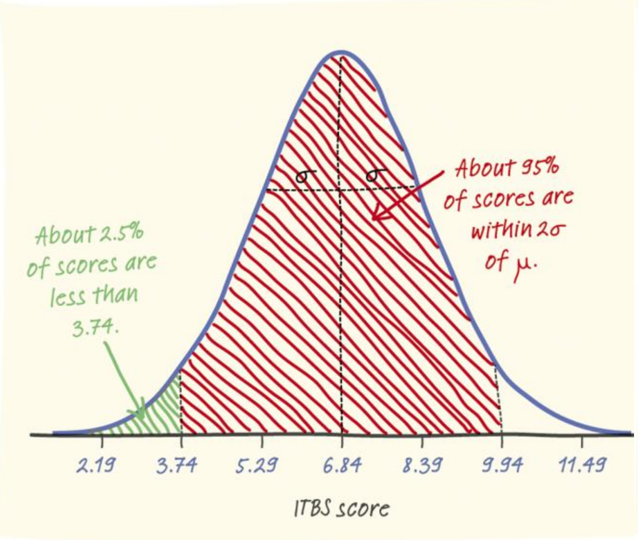
$N\ (\mu, \sigma)$

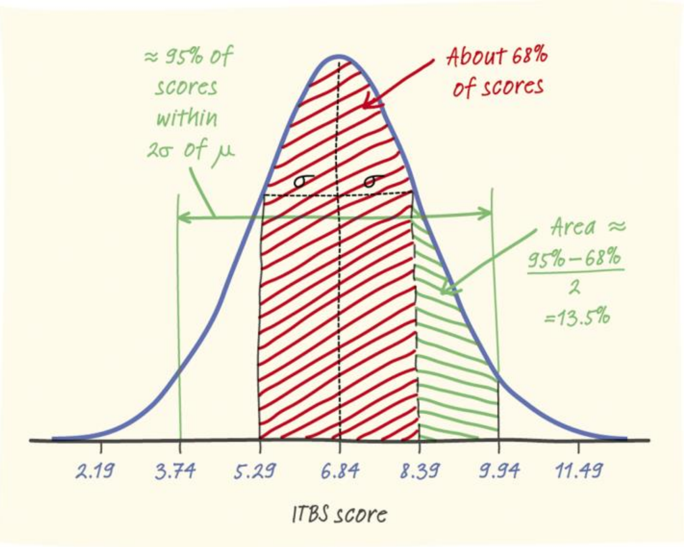
a) Sketch the Normal density curve for this distribution.

b) What percent of ITBS vocabulary scores are less than 3.74?

c) What percent of the scores are between 5.29 and 9.94?







Nov 5, 2021: Warm - UP

1) Given a fairly symmetric distribution that has a mean of 100 and a standard deviation of 15, what are the following z-scores:

- A) for an observation that is 110
- B) for an observation that is 85
- C) for an observation that is 142

2) Using the information from the problem above, what is the value of the observation that has a z-score: a) $z = -2$; b) $z = 1.58$

Nov. 2021: Warm-UP Answers

1) Given a fairly symmetric distribution that has a mean of 100 and a standard deviation of 15, what are the following z-scores:

$$z\ score = \frac{x\ - mean}{standard\ deviation}$$

- A) for an observation that is 110

$$z\ score = \frac{110\ - 100}{15} = 0.667$$

- B) for an observation that is 85

$$z\ score = \frac{85\ - 100}{15} = -1.0$$

- C) for an observation that is 142

$$z\ score = \frac{142\ - 100}{15} = 2.8$$

Nov. 2021: Warm-UP Answers

2) Using the information from the problem above, what is the value of the observation that has a z-score: a) $z\ score = -2$

$$-2 = \frac{x - 100}{15}$$

$$-30 = x - 100$$

$$\therefore\ x = 70$$

b) $z\ score = 1.58$

$$1.58 = \frac{x - 100}{15}$$

$$23.7 = x - 100$$

$$\therefore\ x = 123.7$$

Standard normal Table

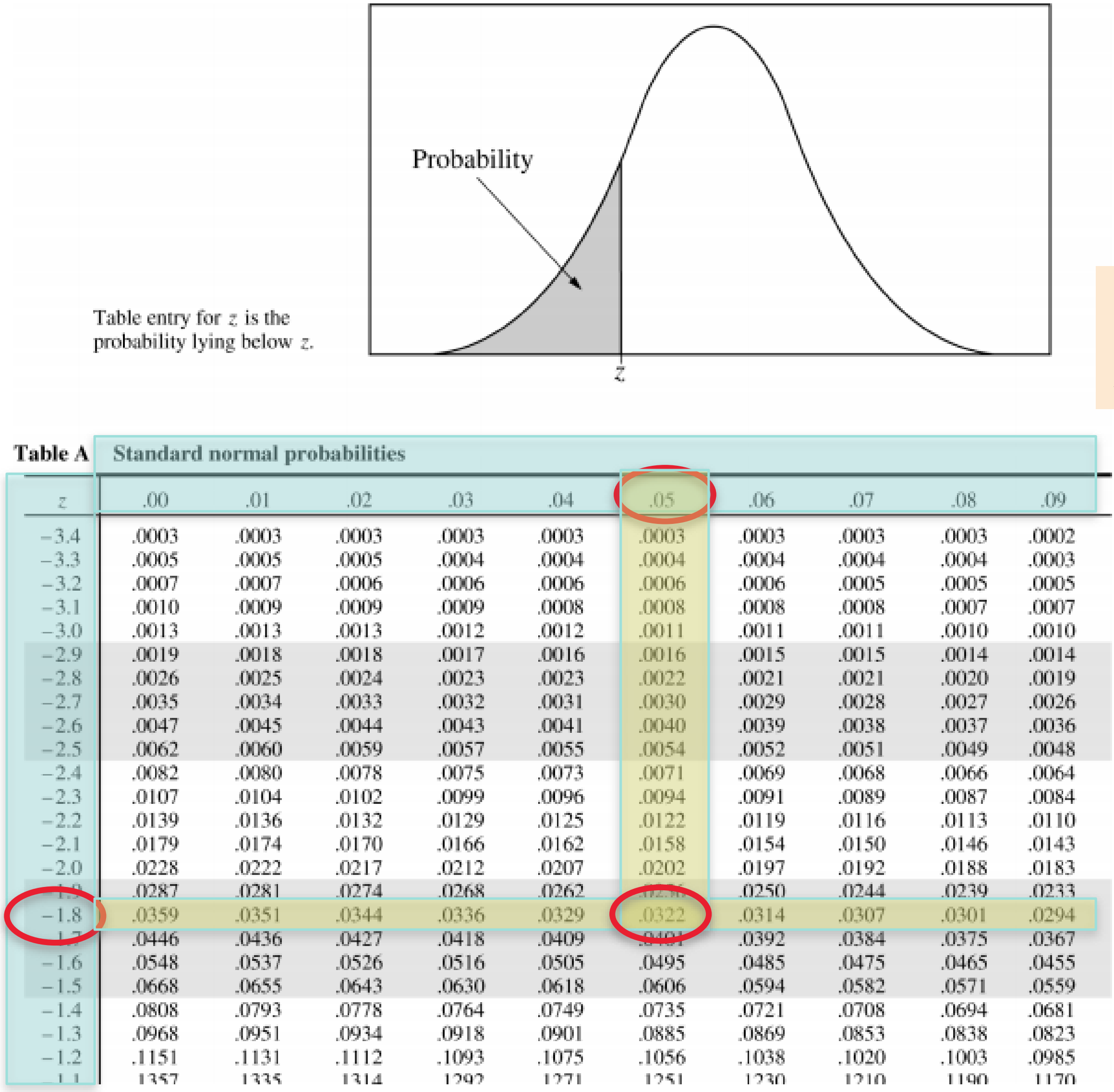


Table entry for *z* is the probability lying below *z*.

Table A Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170

What percentile is a *z-score* of -1.85?

a *z-score* of
-1.85 = 0.0322
So about
3.2% or
3rd percentile

Standard normal Table

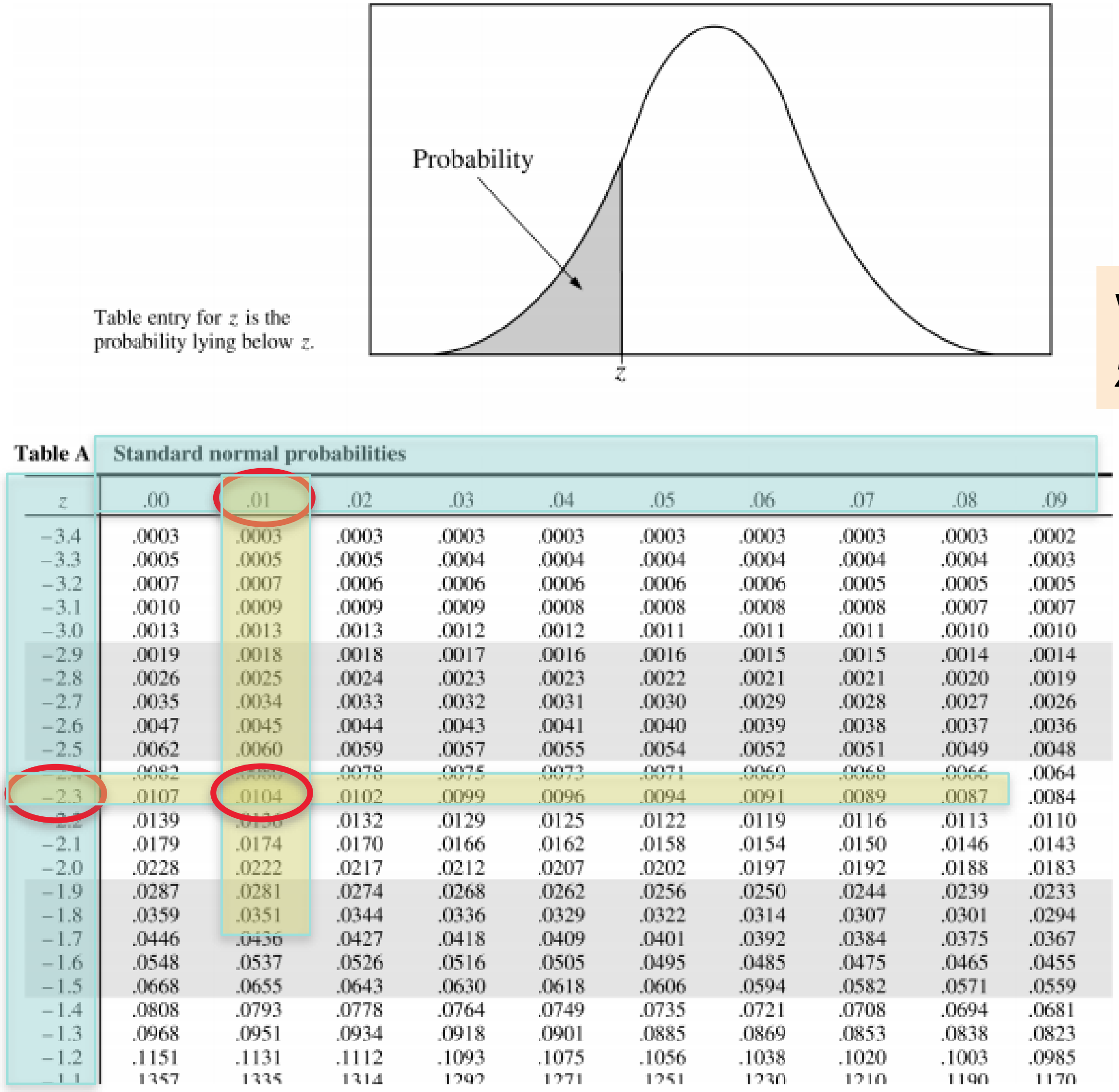


Table entry for *z* is the probability lying below *z*.

Table A Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170

What percentile is a *z-score* of -2.31?

a *z-score* of
-2.31 = 0.0104
So about
1%
1st percentile

Standard normal Table
Finding the z-score

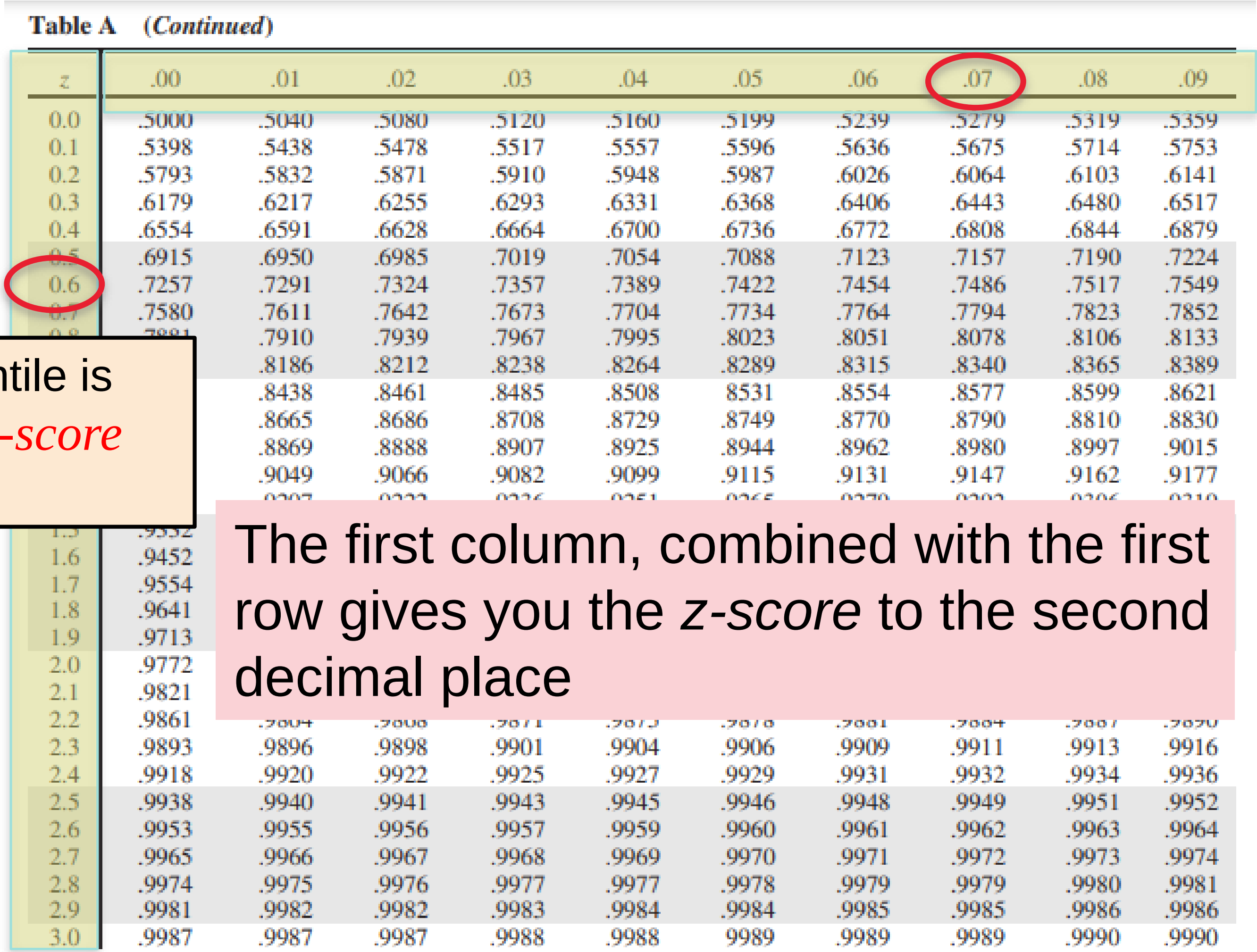


Table A *(Continued)*

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
[illegible]	[illegible]	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
[illegible]	[illegible]	.8438	.8461	.8485	.8508	8531	.8554	.8577	.8599	.8621
[illegible]	[illegible]	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
[illegible]	[illegible]	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
[illegible]	[illegible]	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1.5	.9332	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1.6	.9452	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1.7	.9554	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1.8	.9641	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1.9	.9713	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2.0	.9772	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2.1	.9821	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2.2	.9861	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	9989	.9989	.9989	.9990	.9990

What percentile is equal to a *z-score* of **0.67**?

The first column, combined with the first row gives you the *z-score* to the second decimal place

Standard normal Table
Finding the *percentile* = z-score

What percentile is equal to a *z-score* of **0.67**?

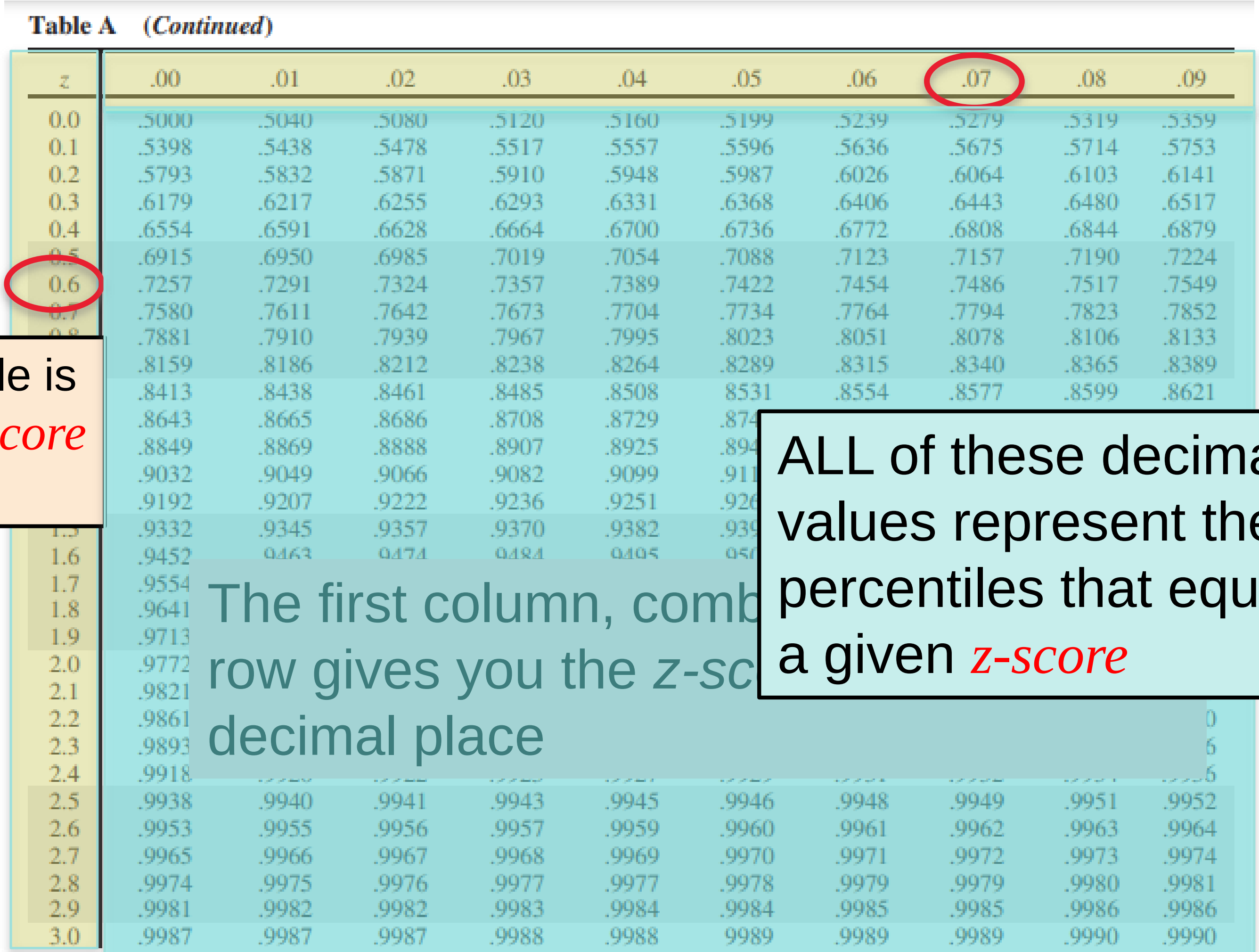


Table A *(Continued)*

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
	.8413	.8438	.8461	.8485	.8508	8531	.8554	.8577	.8599	.8621
	.8643	.8665	.8686	.8708	.8729	.874				
	.8849	.8869	.8888	.8907	.8925	.894				
	.9032	.9049	.9066	.9082	.9099	.911				
	.9192	.9207	.9222	.9236	.9251	.926				
1.5	.9332	.9345	.9357	.9370	.9382	.939				
1.6	.9452	.9463	.9474	.9484	.9495	.950				
1.7	.9554									
1.8	.9641									
1.9	.9713									
2.0	.9772									
2.1	.9821									
2.2	.9861									
2.3	.9893									
2.4	.9918									
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	9989	.9989	.9989	.9990	.9990

The first column, comb
row gives you the *z-sc*
decimal place

ALL of these decimal values represent the percentiles that equal to a given *z-score*

The Standard Normal Table

Because all Normal distributions are the same when we standardize, we can find areas under any Normal curve from a single table.

Definition: **The Standard Normal Table**

Table A is a table of areas under the standard Normal curve. The table entry for each value *z* is the area under the curve to the left of *z*.

Suppose we want to find the proportion of observations from the standard Normal distribution that are less than 0.81.

We can use Z table (z^*):

Z	.00	.01	.02
0.7	.7580	.7[illegible]1	.7642
0.8	.7881	.7910	.7939
0.9	.8159	.8186	.8212

$P(z < 0.81) = .7910$

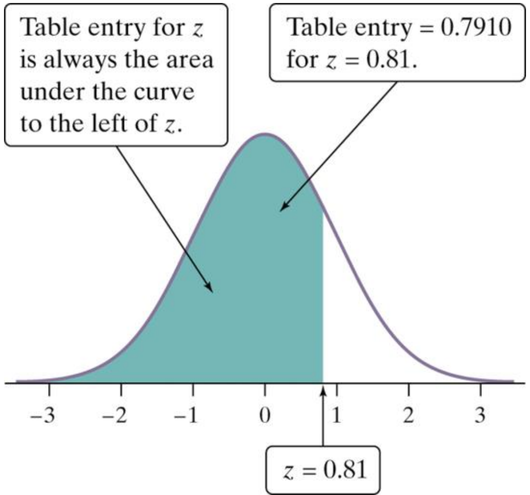


Table entry for z is always the area under the curve to the left of z.

Table entry = 0.7910 for $z = 0.81$.

Example

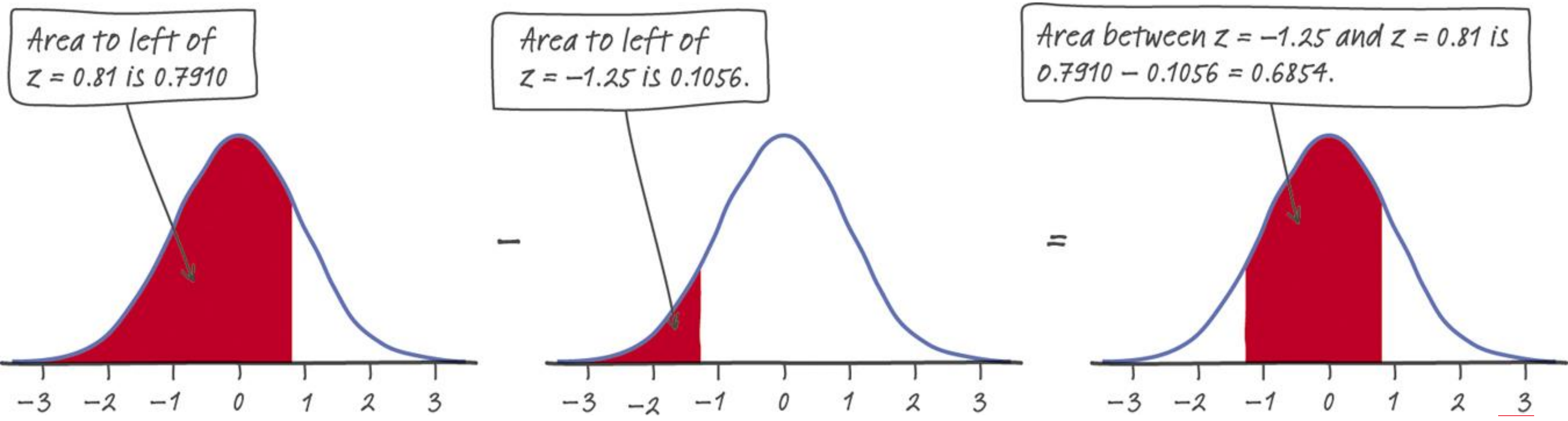
- **Finding Areas Under the Standard Normal Curve**

Find the proportion of observations from the standard Normal distribution that are between -1.25 and 0.81.

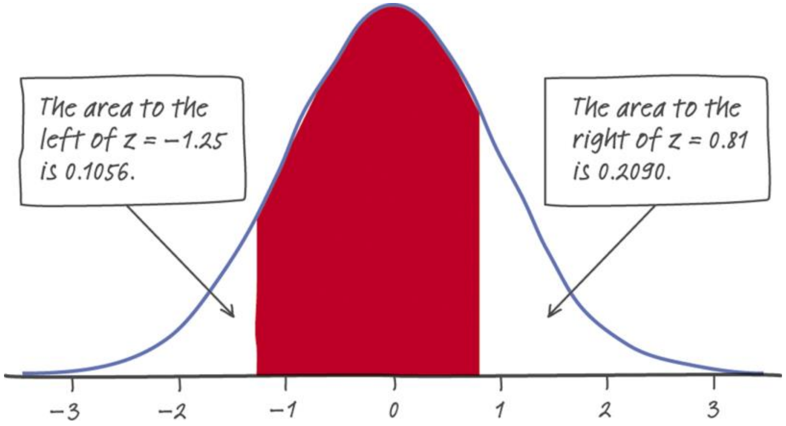
Example

Finding Areas Under the Standard Normal Curve

Find the proportion of observations from the standard Normal distribution that are between -1.25 and 0.81.



Can you find the same proportion using a different approach?



$$1 - (0.1056+0.2090) =$$
$$1 – 0.3146$$
$$\mathbf{= 0.6854}$$

Normal Distribution Calculations

How to Solve Problems Involving Normal Distributions

State: Express the problem in terms of the observed variable *x*.

Plan: Draw a picture of the distribution and shade the area of interest under the curve.

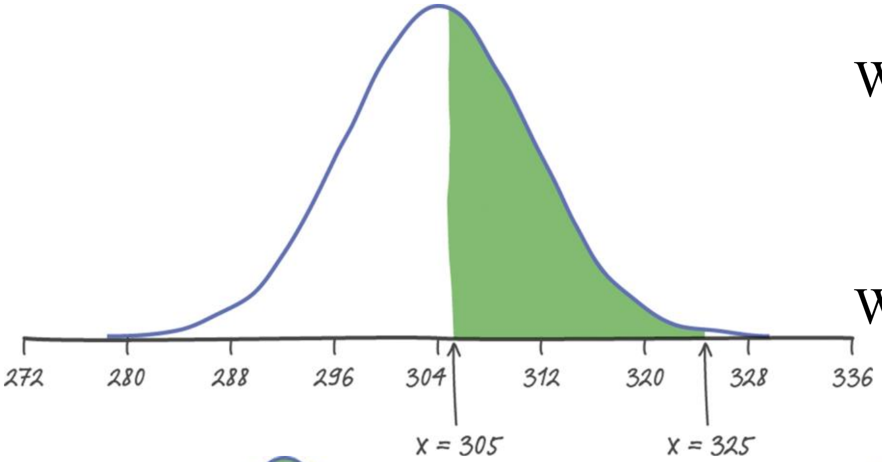
Do: Perform calculations.

- ***Standardize*** *x* to restate the problem in terms of a standard Normal variable *z*.
- ***Use Standard Table*** and the fact that the total area under the curve is 1 to find the required area under the standard Normal curve.

Conclude: Write your conclusion in the context of the problem.

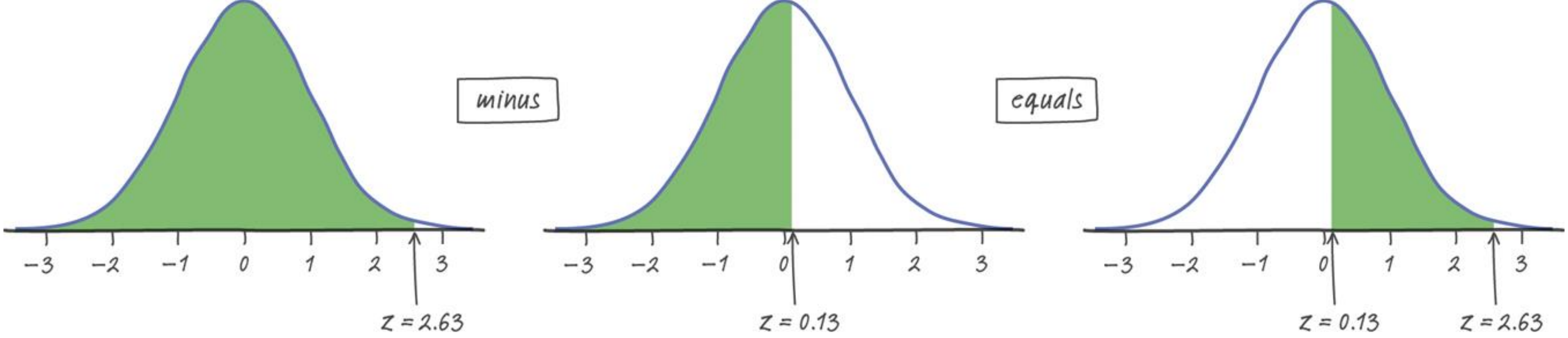
Normal Distribution Calculations

When Tiger Woods hits his driver, the distance the ball travels can be described by *N*(304, 8). What percent of Tiger's drives travel between 305 and 325 yards?



$$\text{When } x = 305,\ z = \frac{305 - 304}{8} = 0.13$$

$$\text{When } x = 325,\ z = \frac{325 - 304}{8} = 2.63$$



Using Table A, we can find the area to the left of z=2.63 and the area to the left of z=0.13.
0.9957 – 0.5517 = 0.4440. About **44%** of Tiger's drives travel between 305 and 325 yards.

Standard Deviation Activity

- Pick up the worksheet and complete all the problems ***without*** using a calculator. Use your mental math and estimation skills.
- Be prepared to share your results.

- **Assessing Normality**

- The Normal distributions provide good models for some distributions of real data. Many statistical inference procedures are based on the *assumption* that the population is approximately Normally distributed. Consequently, we need a strategy for assessing Normality.

✓***Plot the data*.**

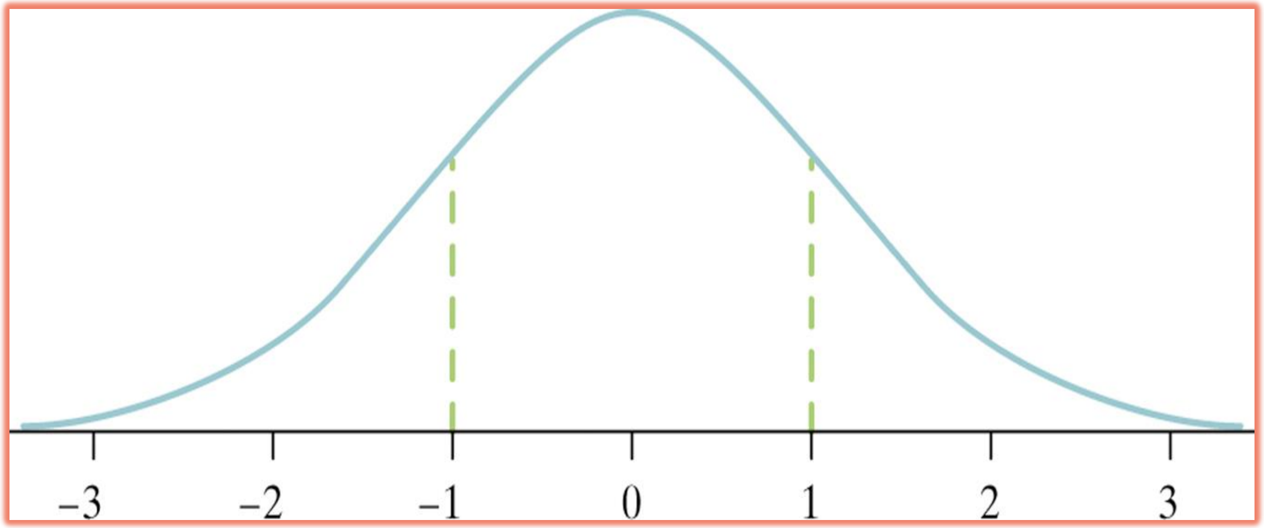
- Make a dotplot, stemplot, or histogram and see if the graph is approximately symmetric and bell-shaped.

✓***Check whether the data follow the 68-95-99.7 rule.***

- Count how many observations fall within one, two, and three standard deviations of the mean and check to see if these percents are close to the 68%, 95%, and 99.7% targets for a Normal distribution.

Assessing Normality

Draw a normal density curve and determine the values for heights that are $\pm 1\sigma, \pm 2\sigma, and \pm 3\sigma$ away from the mean



5’9” 6’0” 6’3” 6’6” 6’9” 7’0” 7’3”

Section 4.5
Normal Distributions

Summary

In this section, we learned that…

- ✓ The **Normal Distributions** are described by a special family of bell-shaped, symmetric density curves called **Normal curves**. The mean μ and standard deviation σ completely specify a Normal distribution *N*(μ,σ). The mean is the center of the curve, and σ is the distance from μ to the change-of-curvature points on either side.
- ✓ All Normal distributions obey the **68-95-99.7 Rule**, which describes what percent of observations lie within one, two, and three standard deviations of the mean.

Summary of Normal Distributions

Summary

In this section, we learned that…

- ✓ All Normal distributions are the same when measurements are standardized. The **standard Normal distribution** has mean µ=0 and standard deviation σ=1.
- ✓ **Standard Normal Table** gives percentiles for the standard Normal curve. By standardizing, we can use Table A to determine the percentile for a given *z*-score or the *z*-score corresponding to a given percentile in any Normal distribution.
- ✓ To assess Normality for a given set of data, we first observe its shape. We then check how well the data fits the **68-95-99.7 rule.** We can also construct and interpret a **Normal probability plot**.