Intro to Probability

Reviewing the basics

Objectives

- Students will find the probability of an event (and the odds of an event).

- Students will understand and be able to compare: (1) Classical probability, (2) Empirical probability, and (3) Subjective probability

- Students will understand and use the vocabulary and notation associated with probability

WARM-UP

- What is the sample space for the outcomes of the sum of rolling 2 dice?
- With your partner, complete 20 trials of rolling 2 dice and record the sum for each
- What is the theoretical probability for rolling each sum?
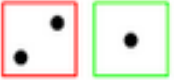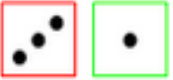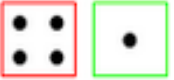- What was the experimental probability for rolling each sum?
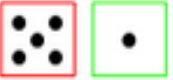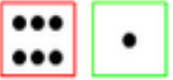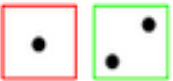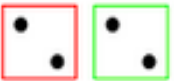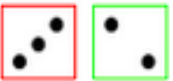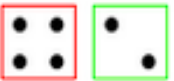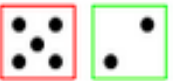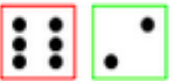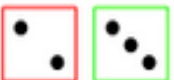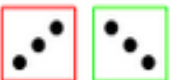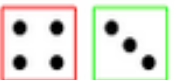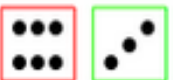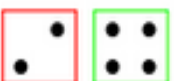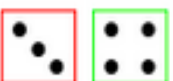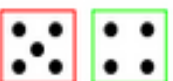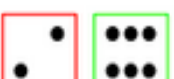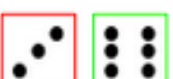
Vocabulary

- **Probability (P)** – is the likelihood that an event will occur, as a value from 0 to 1.
- **Outcomes** – when you do a probability experiment, each _result_ of a single trial is called an outcome.
- **Event** – is an outcome or a collection of outcomes
- **Sample Space** - A list of every possible outcome for a given condition (i.e., rolling dice, or drawing cards, etc.)

What is the sample space for rolling 2 dice?

$$P(2) = \frac{1}{36} = 0.02777 \approx 0.03$$

All possible combinations of 2 dice

36 possible outcomes

WARM-UP

- What is the sample space for the outcomes of rolling 2 dice? **36**

- What is the theoretical probability for rolling each sum? $\frac{x}{36}$

- Since you completed 20 trials, this is you sample space (total possible outcomes)

- What was the experimental probability for rolling each sum? $\frac{x}{20}$

Vocabulary

- **Fundamental Counting Principle** – is used to determine the total number of ways that successive events can occur

- So, if there are *m* ways for one event to occur, and *n* ways for another event to occur, then there are $m \cdot n$ ways for both events to occur

- If you have 5 shirts, 4 pants, and 7 pairs of shoes, you can make __**140**__ outfits.

Mr. L's LUCKY Lottery

If there is time,
ask Mr. L. how YOU
can WIN a guaranteed
A on the FINAL EXAM!

Vocabulary

- **Complement** of an Event P(A') are all of the other outcomes *not* in Event A

Example: Rolling a "5" or "6"

Event A: {5, 6}

Number of ways it can happen: 2

Total number of outcomes: 6

Complement of Event A   Event A

A'   A
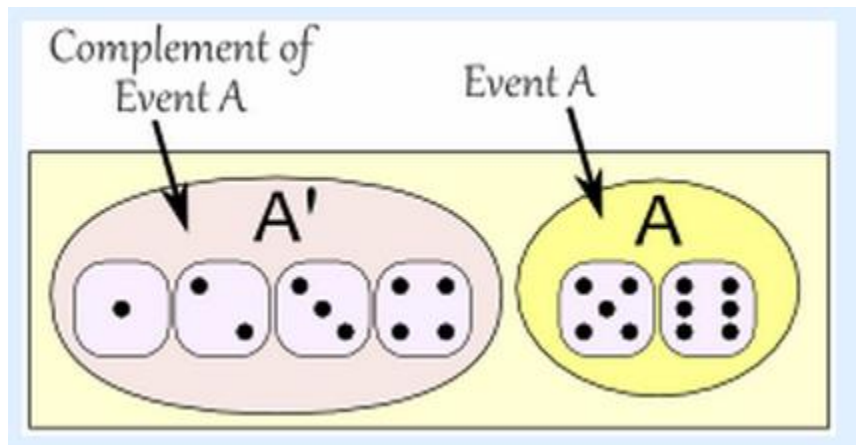
$P(A) = \dfrac{2}{6} = \dfrac{1}{3}$

$P(A') = \dfrac{4}{6} = \dfrac{2}{3}$

The **Complement of Event A** is {1, 2, 3, 4}

$$P(A) + P(A') = 1$$

Types of Probability

- There are 3 types of probability

Theoretical Probability
Experimental Probability
Subjective Probability (not on EOC)

- Let's look at each one individually…

Classical Probability

□ Classical or *Theoretical* Probability is based upon the number of favorable outcomes divided by the total number of outcomes

Example:

■ In the roll of a die, the probability of getting an even number is 3/6 or ½.

■ Notation used:

P (even) $= \frac{3}{6} \ or \ \frac{1}{2}$

How does that work?

- Typically a six-sided die contains the numbers 1, 2, 3, 4, 5, and 6.

- Of those numbers only 2, 4, and 6 are even.

- So, we can set up a ratio of the number of favorable outcomes divided by the total number of outcomes, which is 3/6 or 1/2

Theoretical Probability Formula

If we denote **A** = *desired event*, then
Probability of this event is: P(**A**)

Theoretical Probability :

$$P(\textbf{A}) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

Example # 1

- A box contains **5 green** pens, **3 blue** pens, **8 black** pens and **4 red** pens. A pen is picked at random

- What is the probability that the pen is green?

  There are **5** + **3** + **8** + **4** or **20** pens in the box

$$P \text{ (green)} = \frac{\text{\# green pens}}{\text{Total \# of pens}} = \frac{5}{20} = \frac{1}{4}$$

Experimental (Empirical) Probability

- As the name suggests, **Experimental** (or *empirical*) **Probability** is based upon repetitions of an actual experiment.

Example:

If you toss a coin 10 times and record heads for 8 trials, then the experimental probability of landing on heads was

$$P(heads) = \frac{8}{10} = \frac{4}{5}$$

Experimental Probability Formula

- Experimental Probability:

$$P(E) = \frac{\text{Number of favorable outcomes}}{\text{Total number trials}}$$

Example #2

- In an experiment a coin is tossed **15** times. The recorded outcomes were: **6 heads** and **9 tails**. What was the experimental probability of the coin being heads?

P (heads) = $\dfrac{\text{\# Heads}}{\text{Total \# Tosses}}$ = $\dfrac{6}{15}$ = $\dfrac{2}{5}$

Subjective Probability

- **Subjective probability** describes an individual's personal judgement about how likely a particular event is to occur. It is not based on any precise computation but is often a reasonable assessment based upon given knowledge.

- It is still expressed within the scale from 0 (impossible) to 1 (certain).
- NOTE: This is not covered on the EOC

Sample Space of 2 Dice

All possible combinations of 2 dice

Experimental Prob.

"I'm confident that my next roll is going to equal **LUCKY 7**!"

Subjective Prob.

Rolled the dice 10 times:
1st: 2, 5
2nd: 3, 4
3rd: 1, 3
4th: 5, 5
5th: 6, 1
6th: 3, 6
7th: 5, 4
8th: 4, 4
9th: 1, 2
10th: 4, 3

P (sum of 7) = $\frac{6}{36}$ = $\frac{1}{6}$

Theoretical Prob.

P (sum 7) = $\frac{4}{10}$

Compound Events

- The UNION ($\cup$) or INTERSECTION ($\cap$) of two events (or more than two events) is called a **compound event**
- If P(A) = probability that event A occurs
- If P(B) = probability that event B occurs
- The **U**NION ($\cup$) of two event is the same as finding P (A *or* B) = P(A $\cup$ B) ;
- The INTERSECTION ($\cap$) of two event is the same as finding P (A *and* B) = P(A $\cap$ B)

Compound Events

- The **addition rule** is a result used to determine the probability that event A or event B occurs or both occur; UNION (∪):

- $P(A \cup B) = P(A) + P(B) - P(A \text{ and } B)$

- If the events do not share any outcomes in common (*mutually exclusive*), then the P(A or B) is simply $P(A \cup B) = P(A) + P(B)$

Vocabulary for Probability

- Two events are **mutually exclusive** (or *disjoint*) if it is impossible for them to occur together.

- Example: Drawing one card from a deck that is both an Ace and a King

- Notation: P (A *and* B) = P$(A \cap B)$

- P (A *and* B) = 0
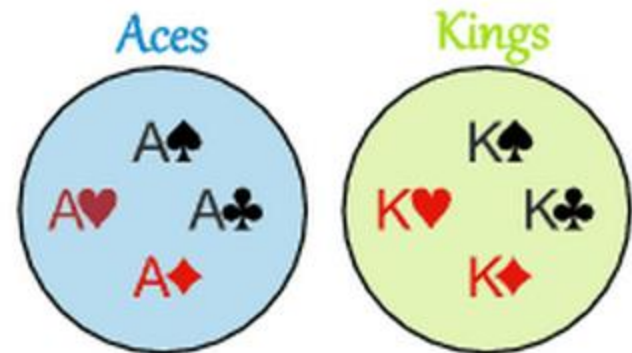
Compound Event Example

- Suppose we wish to find the probability of drawing either a face card (K, Q, J) or a seven in a single draw from a pack of 52 cards:

- We define the events:

- Event A = draw a face card; and
  Event B = draw a seven

- so P(Face card or a seven) can be written as the **U**nion of A or B: $P(A \cup B)$

Compound Events (cont.)

- Note that face cards and sevens are…
- **mutually exclusive,** so
- $P(\text{Face} \bigcup Seven) = P(A) + P(B)$

- Since there are 12 face cards in the pack and 4 sevens, we have:
- $P(\text{Face} \bigcup Seven) = P(A) + P(B) =$
$$\frac{12}{52} + \frac{4}{52} = \frac{16}{52} = \frac{4}{13}$$

Compound Event Example

- Suppose we wish to find the probability of drawing either a king or a red card in a single draw from a pack of 52 cards:

- We define the events:

- Event A = draw a king; and
  Event B = draw a red card

- so P(King or a Red) can be written as the **U**nion of A or B: $P(A \cup B)$

Compound Events (cont.)

- $P(\text{King} \cup Red)$ are NOT mutually exclusive

- so $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

- Since there are 4 kings in the pack and 26 red cards, but 2 cards are both a king and red, we have:

- $P(\text{King} \cup Red) = P(A) + P(B) - P(A \text{ and } B) =$

$$\frac{4}{52} + \frac{26}{52} - \frac{2}{52} = \frac{28}{52} = \frac{7}{13}$$

- So, the probability of drawing either a king or a red card is $\frac{7}{13}$

Compound Events

- The multiplication rule is a result used to determine the probability that two events, A and B, both occur; INTERSECTION (∩)

- Notation: P (A *and* B) = $P(A \cap B)$

- For **independent events**, that is events which have no influence on one another, the rule is P (A *and* B) = P (A)$\cdot$P(B)

Compound Events

- Given a 6-sided die and a fair coin, what is the probability of rolling a 5 or 6 and getting tails?

- We define the events:

- Event A = rolling a 5 or 6; and
  Event B = coin lands on tails

- Find the P (A *and* B) = P (A)$\cdot$ P(B)

$$P(A \cap B) = \frac{2}{6} \cdot \frac{1}{2} = \frac{2}{12} = \frac{1}{6}$$

Finding the **U**nion of Events

The **addition rule** is a result used to determine the probability that event A or event B occurs or both occur; UNION (∪):

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

(the sum of each event's probability minus their intersection)

If the events do not share any outcomes in common (*mutually exclusive*), then the union P(A or B) is simply $P(A \cup B) = P(A) + P(B)$

Conditional Probability

- Sometimes the probability of an event must be computed using the knowledge that some other event has happened (or is happening, or will happen – the timing is not important). This type of probability is called *conditional probability*.

Conditional Probability

- P(A | B) = the (conditional) probability that event A will occur given that event B has occurred already

- The usual notation for "event A occurs given that event B has occurred" is "A | B" (A given B). The symbol | is a vertical line and does not imply division. P(A | B) denotes the probability that event A will occur given that event B has occurred already.

Compound Events

- Two events *A* and *B* are called **independent events** if knowledge about the occurrence of one of them has no effect on the probability of the other one, that is, if

- $P(B \mid A) = P(B)$, or equivalently

- $P(A \mid B) = P(A)$.

Independent events

- We define the events:
- Event A = rolling a 5 or 6; and
  Event B = coin lands on tails

- **$P(B \mid A) = P(B)$:**

- $P\ (tails \mid roll\ 5\ or\ 6) = \frac{1}{2}$

- **$P(A \mid B) = P(A)$:**

- $P\ (roll\ 5\ or\ 6 \mid tails) = \frac{2}{6} = \frac{1}{3}$

Conditional Probability

- The probability of event *B*, computed on the assumption that event *A* has happened, is called the **conditional probability of *B*, given *A*,** and is denoted ***P*(*B* | *A*).**

- What is the probability of drawing an Ace from a deck of 52 cards?

- What is the probability of drawing an Ace from a deck, given that you already drew an Ace?

Conditional Probability

- The **conditional probability of *B*, given *A*,** and is given by

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A \text{ and } B)}{P(A)}.$$

P (Draw an Ace , given you drew an Ace)

$$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{4}{52} \cdot \frac{3}{51}}{\frac{4}{52}} = \frac{1}{17}$$

Dependent events

□ Two events are called **dependent events** if the occurrence of one affects the occurrence of the other.

□ P (A *and* B) = P (A)· $\mathrm{P}(B\,|\,A)$ *or*

□ P (A ∩ B) $= \dfrac{4}{52} \cdot \dfrac{1}{17} = \dfrac{\mathbf{1}}{\mathbf{221}}$

Warm-UP

From the sample space
S = {2, 3, 4, 5, 6, 7, 8, 9}, a single number is to be selected randomly. Given the events
A: selected number is odd, and
B: selected number is a multiple of 3.

find each probability.
a) $P(B)$
b) $P(A$ and $B)$
c) $P(B \mid A)$

Example Solutions

- *A:* selected number is odd, and
- *B:* selected number is a multiple of 3.

a) *B* = {3, 6, 9}, so *P*(*B*) = 3/8

b) *P*(*A* and *B*) = {3, 5, 7, 9} & {3, 6, 9} = {3, 9}, so

$$P(A \text{ and } B) = 2/8 = ¼$$

or $P(A \cap B) = P(A) \cdot P(B| A) = \frac{4}{8} \cdot \frac{1}{2} = \frac{1}{4}$

c) The given condition *A* reduces the sample space to {3, 5, 7, 9}, so *P*(*B* | *A*) = 2/4 = 1/2

Odds

Odds

- Another way to describe the chance of an event occurring is with **odds**. The odds in **favor** of an event is the ratio that compares the number of ways the event **can** occur to the number of ways the event **cannot** occur.

- We can determine odds using the following ratios:

Odds in Favor = $\dfrac{\text{number of successes}}{\text{number of failures}}$

Odds against = $\dfrac{\text{number of failures}}{\text{number of successes}}$

Example

- Suppose we play a game with 2 number cubes.

- If the sum of the numbers rolled is 6 or less – **you win!**

- If the sum of the numbers rolled is not 6 or less – **you lose**

In this situation we can express odds as follows:

Odds in favor = $\dfrac{\text{numbers rolled is 6 or less}}{\text{numbers rolled is not 6 or less}} \quad \dfrac{15}{21} = \dfrac{5}{7}$

Odds against = $\dfrac{\text{numbers rolled is not 6 or less}}{\text{numbers rolled is 6 or less}} \quad \dfrac{21}{15} = \dfrac{7}{5}$

Example

- A bag contains 5 yellow marbles, **3 white** marbles, and 1 black marble. What are the odds drawing a **white** marble from the bag?

Odds in favor = $\dfrac{\text{number of white marbles}}{\text{number of non-white marbles}}$ $\dfrac{3}{6}$

Odds against = $\dfrac{\text{number of non-white marbles}}{\text{number of white marbles}}$ $\dfrac{6}{3}$

Therefore, **the odds for are 1:2**
and **the odds against are 2:1**

Comments

- On the next couple of slides are some practice problems…The answers are on the last slide…

- Do the practice and then check your answers…If you do not get the same answer you must question what you did…go back and problem solve to find the error…

- If you cannot find the error bring your work to me and I will help…

Your Turn - Probability

▫ Find the probability of randomly choosing a **specific** marble from the given bag of red and white marbles.

1. Number of **red marbles**   16
   Total number of marbles   64   P(red)

2. Number of **red marbles**   8
   Total number of marbles   40   P(white)

3. Number of white marbles   7
   Total number of marbles   20   P(white)

4. Number of white marbles   24
   Total number of marbles   32   P(red)

                        4b)   P(red | white)

Your Turn - Odds

- Find the **favorable** odds of choosing the indicated letter from a bag that contains the letters in the name of the given state.

5. S; Mississippi $\dfrac{4}{7}$

6. N; Pennsylvania $\dfrac{3}{9} = \dfrac{1}{3}$

7. A; Nebraska $\dfrac{2}{6} = \dfrac{1}{3}$

8. G; Virginia $\dfrac{1}{7}$

Your Turn

- You toss a six-sided number cube 20 times. For twelve of the tosses the number tossed was 3 or more.

9. What is the experimental probability that the number tossed was 3 or more?

10. What are the favorable odds that the number tossed was 3 or more?

Your Turn Solutions

1. ¼
2. 4/5
3. 13/20
4. ¼
5. 4/7

6. 3/9 or 1/3
7. 2/6 or 1/3
8. 1/7
9. 3/5
10. 3/2