Warm-UP (Oct 24/25, 2022)

- Take a copy of the half sheet, A Physician's Thinking and carefully read the scenario that begins the "To facilitate early detection of breast cancer…"
- Write down in your notes/warm-ups the **specified population** in the scenario
- Write down specifically what you are trying to find.
- Wait to obtain the $P(A \mid B)$ from Mr. L…Work it out and be ready to share your process for finding a solution

Warm-UP

- The specified population in the scenario:

  Asymptomatic women aged 40 to 50 who participate in mammography screening

- What you are trying to find?

  What is the probability that **she** actually has breast cancer?

*Given a woman from this pop. that **tests positive**, what is the probability that **she** actually has breast cancer?*

What is the probability that a woman who tests positive for breast cancer actually has breast cancer?

A. 99%

B. 93%

C. 90%

D. 50%

E. 9%

Warm-UP: *One possible* process for the solution

- Consider any 1000 women from the population
- Given 0.8% incident rate for breast cancer
- of these 1000: 8 w/ cancer 992 without
- test results:

8 w/ cancer		992 without	
0.9 Pos	0.1 FALSE neg.	0.93 Neg	0.07 FALSE pos.
≈ 7	≈ 1	≈ 922	≈ 70

Therefore, only 7 out 77 women who had a positive result should expect to actually have breast cancer ($\approx 9\%$)

Chapter 6: Probability: What are the Chances?

Section 6.1- Chance Experiments & Events

Probability Rules

Adapted from The Practice of Statistics, STARNES YATES, MOORE, 4th edition – For AP*

Chapter 6
Probability: What Are the Chances?

- 6.1 Chance Experiments & Events
- **6.2 Probability Rules**
- 6.3 Conditional Probability and Independence

Warm-Up Oct., 2022: Consider flipping a coin 3 times

Avery tosses a fair coin three times.

a) Draw a tree diagram to show all the possible outcomes.

b) Find the probability of getting:
   (i) Three tails.
   (ii) Exactly two heads.
   (iii) At least two tails.

Section 6.2
Probability Rules

Learning Objectives

After this section, you should be able to…

- ✓ DESCRIBE chance behavior with a probability model
- ✓ DEFINE and APPLY basic rules of probability
- ✓ DETERMINE probabilities from two-way tables
- ✓ CONSTRUCT Venn diagrams and DETERMINE probabilities

Addition Rule: to find $P(\mathrm{A} \cup \mathrm{B})$

Multiplication Rule: *to find* $P(A \cap B)$

- Addition Rule : The probability of the Union of two events is found by adding their separate probabilities, and then subtracting their intersection:
- $P(\mathrm{A} \cup \mathrm{B}) = P(A) + P(B) - P(A \cap B)$
- Multiplication Rule: The probability of the i∩tersection of two events is found by multiplying the probability of one event times the conditional probability of the other:
- $P(\mathrm{A} \cap \mathrm{B}) = \mathrm{P(A)} \cdot P(B|A)$ or $\mathrm{P(B)} \cdot P(A|B)$

Probability Models

In Section 5.1, we used simulation to imitate chance behavior. Fortunately, we don't have to always rely on simulations to determine the probability of a particular outcome.

Descriptions of chance behavior contain two parts:

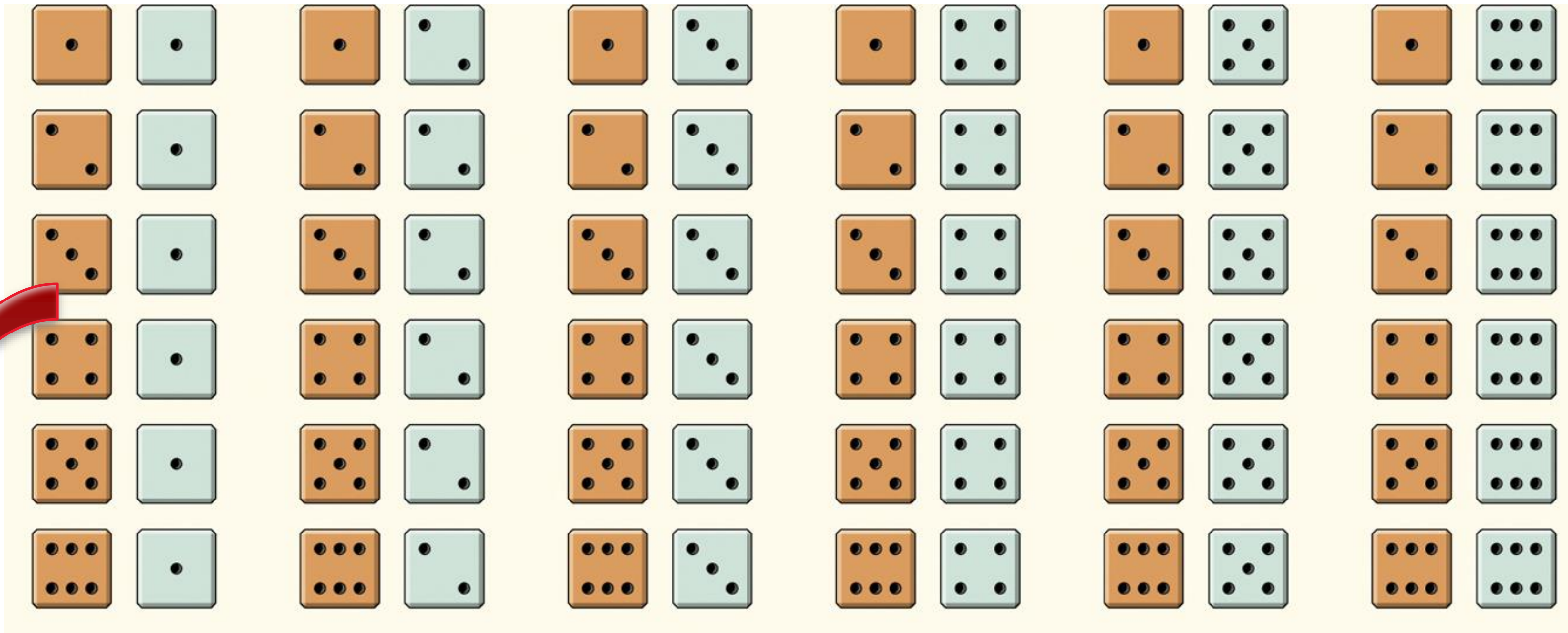
Definition:

The **sample space *S*** of a chance process is the set of all possible outcomes.

A **probability model** is a description of some chance process that consists of two parts: a sample space *S* and a probability for each outcome (probability distribution). The sum of the outcomes for the distribution must equal 1.

Example: Roll the Dice

Give a probability model for the chance process of rolling two fair, six-sided dice – one that's red and one that's green.



Sample Space 36 Outcomes

Since the dice are fair, each outcome is equally likely. Each outcome has probability 1/36.

Probability Models

Probability models allow us to find the probability of any collection of outcomes.

Definition:

An **event** is any collection of outcomes from some chance process. That is, an event is a subset of the sample space. Events are usually designated by capital letters, like *A, B, C,* and so on.

If *A* is any event, we write its probability as P(*A*).

In the dice-rolling example, suppose we define event *A* as "sum is 5."

There are 4 outcomes that result in a sum of 5.
Since each outcome has probability 1/36, P(*A*) = 4/36.

Suppose event *B* is defined as "sum is not 5." What is P(*B*)? *P*(*B*) = 1 – 4/36
= 32/36

Warm-Up: Consider flipping a coin 3 times

Brennen tosses a fair coin three times.

a) Draw a tree diagram to show all the possible outcomes.

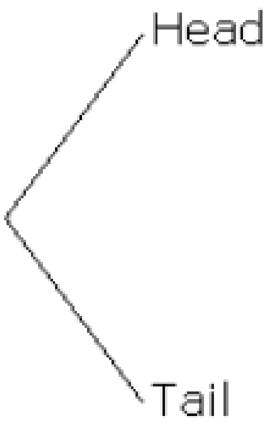
b) Find the probability of getting:
   (i) Three tails.
   (ii) Exactly two heads.
   (iii) At least two tails.

Consider flipping a coin 3 times

Solution:

a) A tree diagram of all possible outcomes.

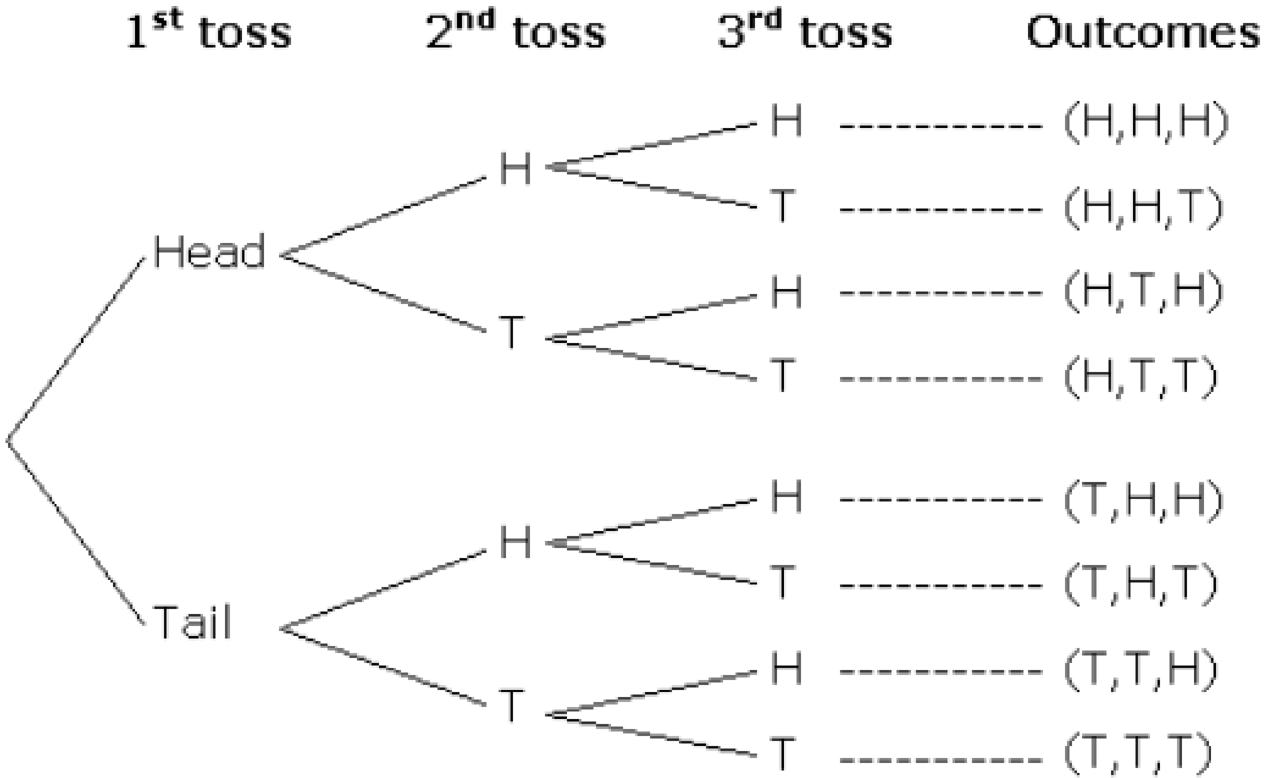
1st toss



Consider flipping a coin 3 times

Solution:

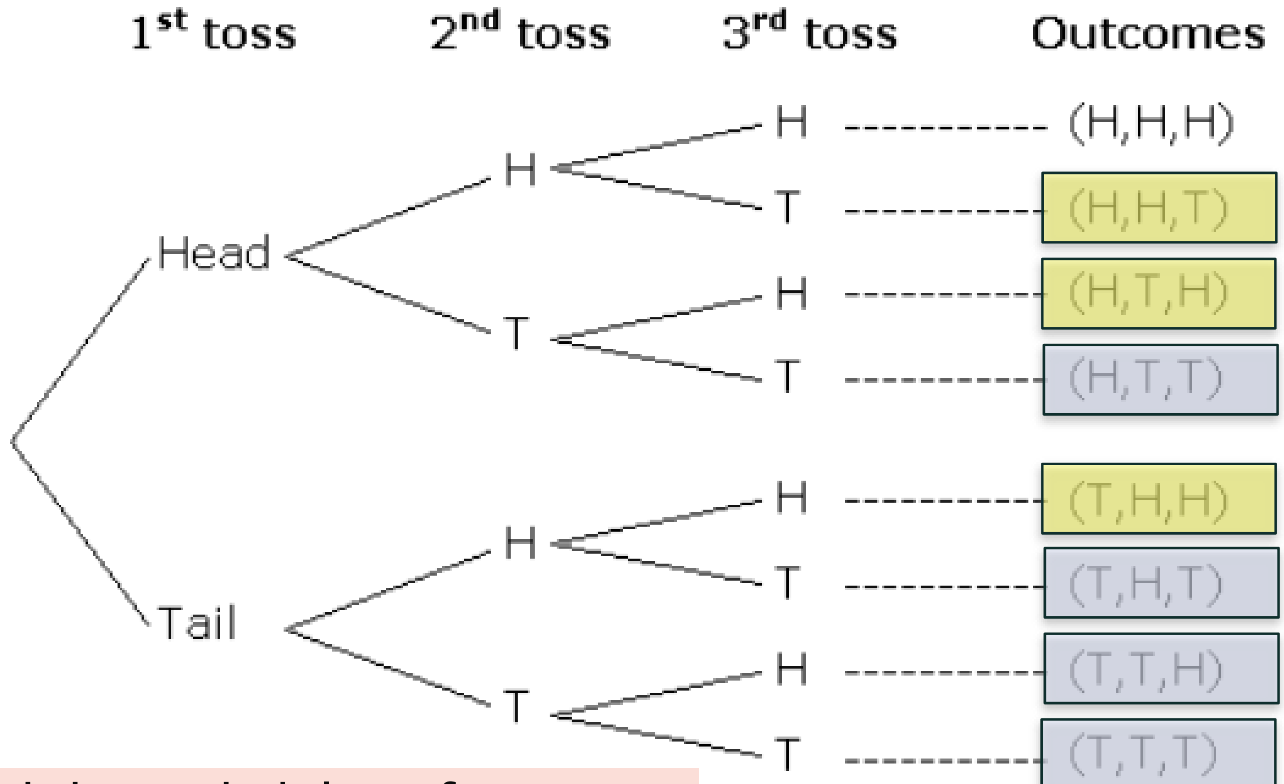
a) A tree diagram of all possible outcomes.



Consider flipping a coin 3 times

Solution:

a) A tree diagram of all possible outcomes.



Find the probability of getting:

(iii) At least two tails $= \frac{4}{8} = \mathbf{0.500}$

(two tails or *three tails)*

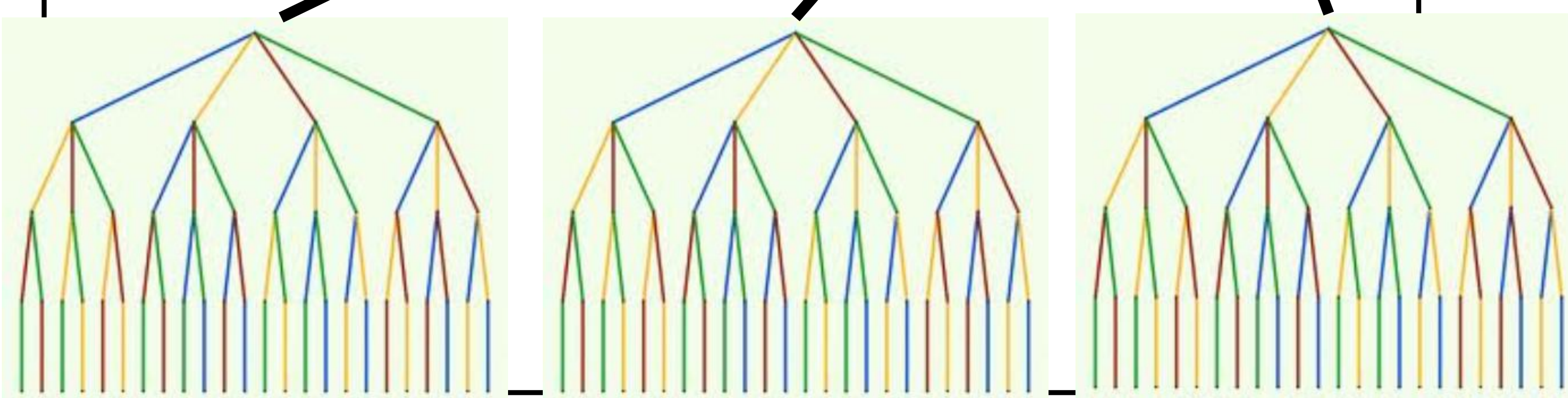
+

Tree diagrams: Graphical display to help organize all possible outcomes

How many ways can you order **5 cards**?

Any of the 5 cards ordered 1st

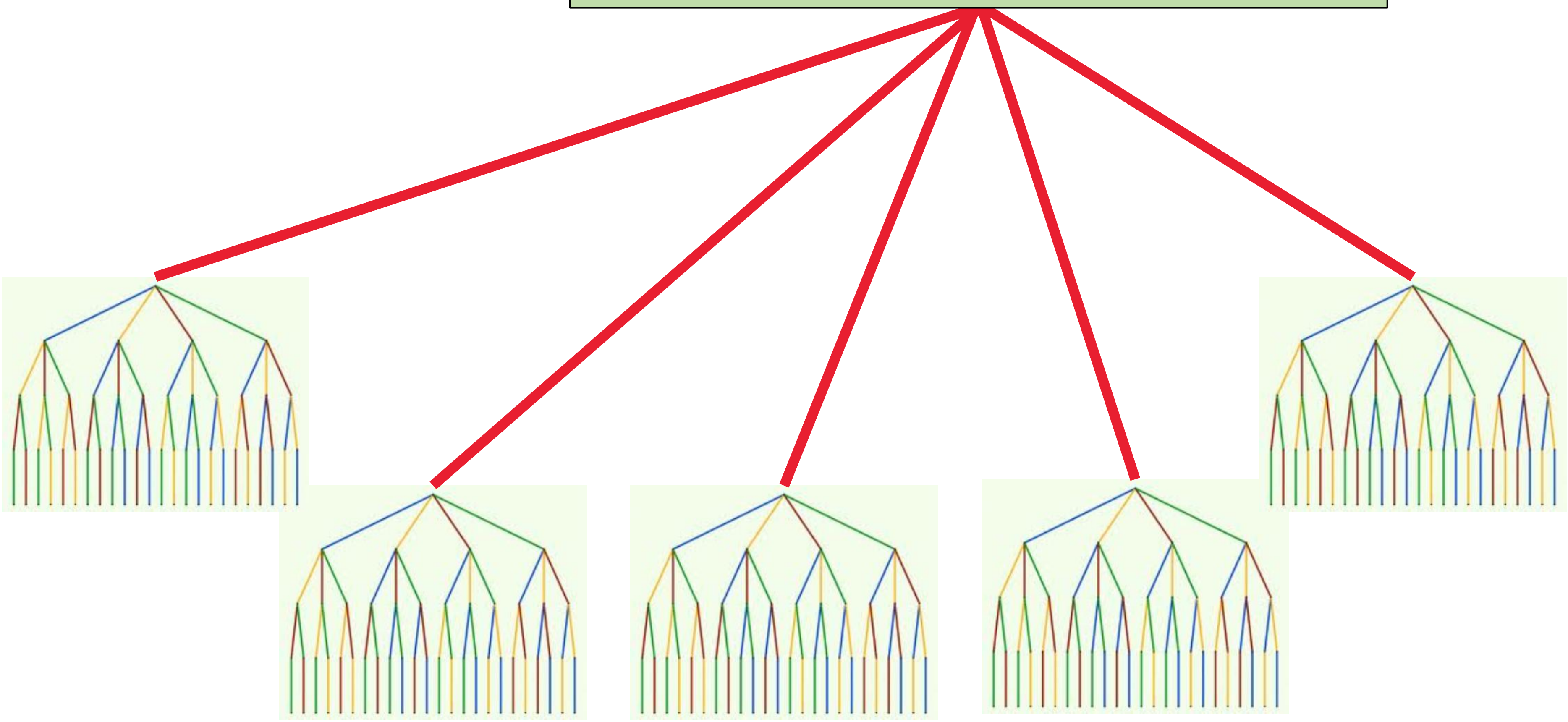
$$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = \mathbf{120}$$



Tree Diagram:

How many ways can you order **5 cards**?

$$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = \mathbf{120}$$

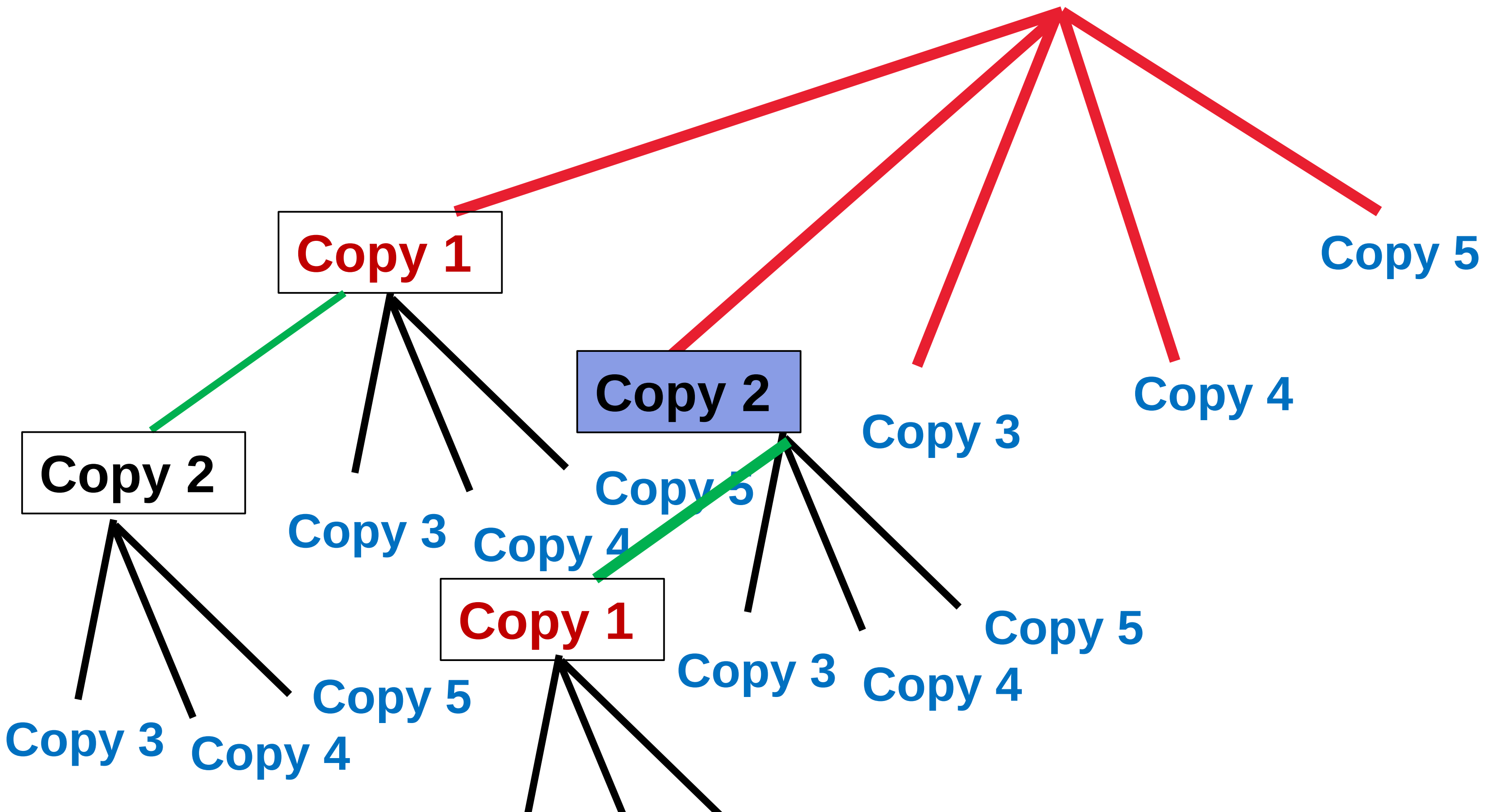


HW Problem #6.7

Selecting a certain textbook

Copies 1 and 2 (hardcover)

Copies 3, 4, and 5 (softcover)



Partner WS: Odds or Evens

Get a pair of dice, then play the games and complete questions 1 to 4

Name: ______________________________ Block: _____ Date: ______

Unit 4: Day 2: Odds or evens, who will win?

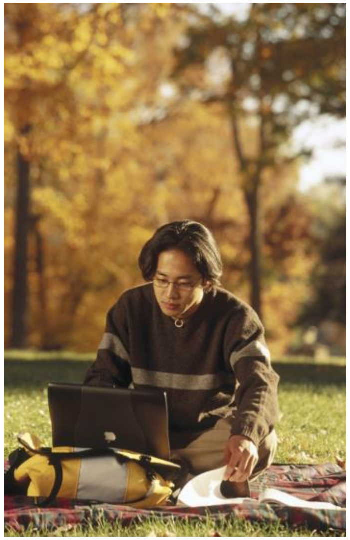
We're going to play a game to answer this question. You and your partner must decide who will be "Odds" and who will be "Evens". Then you will roll two dice and **multiply** the numbers. If the product is odd, the odds person wins and vice versa for evens. Play 20 times, keeping track of how many wins each person has.

1. How many times did the odds win? __________

Example: Distance Learning

Distance-learning courses are rapidly gaining popularity among college students. Randomly select an undergraduate student who is taking distance-learning courses for credit and record the student's age. Here is the probability model:

Age group (yr):	18 to 23	24 to 29	30 to 39	40 or over
Probability:	0.57	0.17	0.14	0.12



(a) Show that this is a legitimate probability model.

Each probability is between 0 and 1 and
0.57 + 0.17 + 0.14 + 0.12 = 1

(b) Find the probability that the chosen student is not in the traditional college age group (18 to 23 years).

$$\boldsymbol{P(\text{not 18 to 23 years}) = 1 - P(\text{18 to 23 years})}$$
$$\boldsymbol{= 1 - 0.57 = 0.43}$$

Basic Rules of Probability

All probability models must obey the following rules:

- **The probability of any event is a number between 0 and 1.**
- **All possible outcomes together must have probabilities whose sum is 1.**
- **If all outcomes in the sample space are equally likely, the probability that event *A* occurs can be found using the formula**

$$P(A) = \frac{\text{number of outcomes corresponding to event } A}{\text{total number of outcomes in sample space}}$$

- **The probability that an event does not occur is 1 minus the probability that the event does occur.**
- **If two events have no outcomes in common, the probability that one or the other occurs is the sum of their individual probabilities.**

Definition:

Two events are **mutually exclusive (disjoint)** if they have no outcomes in common and so can never occur together.

Basic Rules of Probability

- For any event A, $0 \le P(A) \le 1$.
- If S is the sample space in a probability model,

  $$P(S) = 1.$$

- In the case of equally likely outcomes,

  $$P(A) = \frac{\text{number of outcomes corresponding to event } A}{\text{total number of outcomes in sample space}}$$

- **Complement rule:** $P(A^C) = 1 - P(A)$
- **Addition rule for mutually exclusive events:** If A and B are mutually exclusive,

  $$P(A \text{ or } B) = P(A) + P(B).$$

Two-Way Tables and Probability

When finding probabilities involving two events, a two-way table can display the sample space in a way that makes probability calculations easier.

Consider the example for students with pierced ears. Suppose we choose a student at random. Find the probability that a chosen student

	Pierced Ears?		
Gender	**Yes**	**No**	**Total**
Male	19	71	**90**
Female	84	4	**88**
Total	**103**	**75**	**178**

(a) has pierced ears.

(b) is a male with pierced ears.

(c) is a male or has pierced ears.

Define events *A*: is male and *B*: has pierced ears.

(a) Each student is equally likely to be chosen. 103 students have pierced ears. So, *P*(pierced ears) = *P*(*B*) = 103/178.

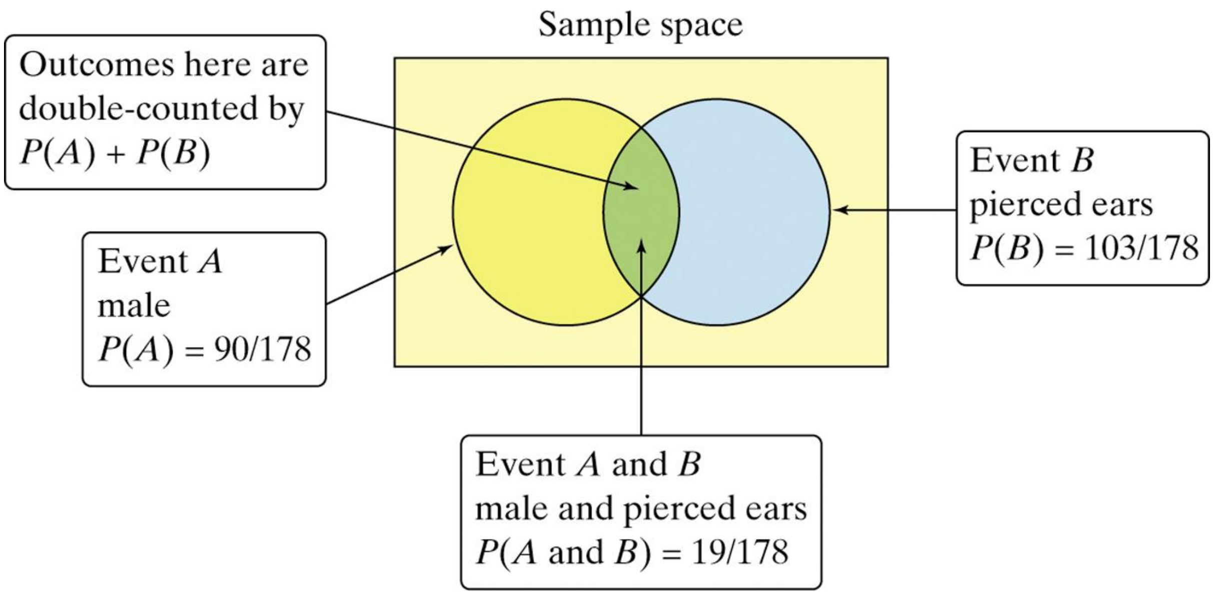
(b) We want to find *P*(male and pierced ears), that is, *P*(*A* and *B*). Look at the intersection of the "Male" row and "Yes" column. There are 19 males with pierced ears. So, *P*(*A* and *B*) = 19/178.

(c) We want to find *P*(male *or* pierced ears), that is, *P*(*A* or *B*). There are 90 males in the class and 103 individuals with pierced ears. However, 19 males have pierced ears – don't count them twice! *P*(*A* or *B*) = (19 + 71 + 84)/178. So, *P*(*A* or B) = 174/178

Two-Way Tables and Probability

Note, the previous example illustrates the fact that we can't use the basic addition rule for mutually exclusive events unless the events have no outcomes in common.

The **Venn diagram** below illustrates why.



General Addition Rule for Two Events

If A and B are any two events resulting from some chance process, then

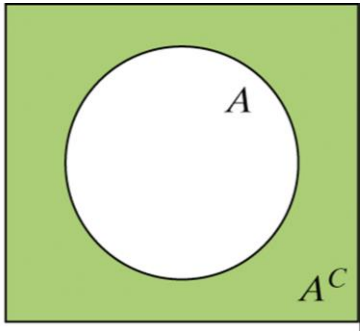
$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Probability Rules

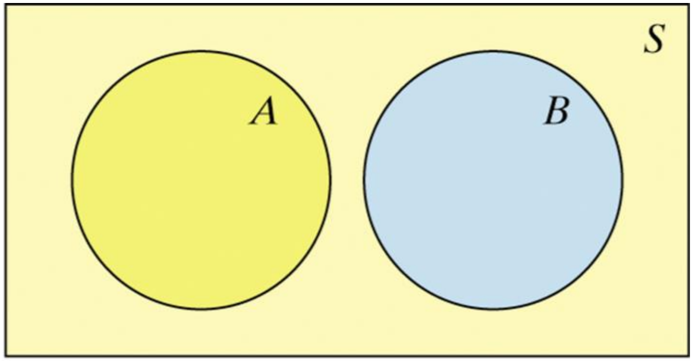
Venn Diagrams and Probability

Because Venn diagrams have uses in other branches of mathematics, some standard vocabulary and notation have been developed.

The complement A^C contains exactly the outcomes that are not in A.

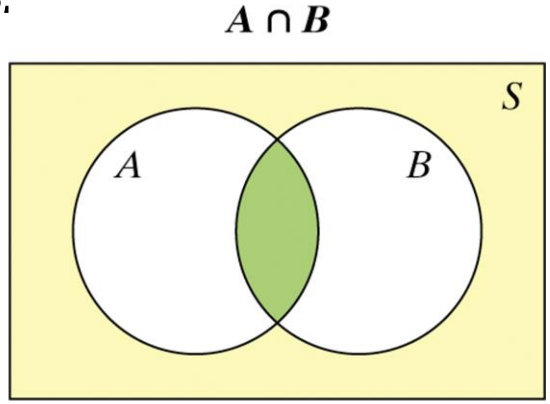


The events A and B are mutually exclusive (disjoint) because they do not overlap. That is, they have no outcomes in common.

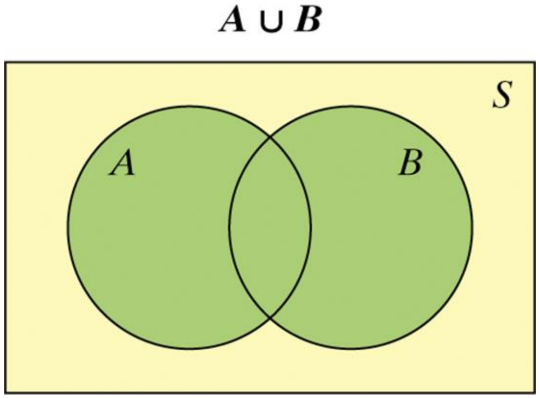


Venn Diagrams and Probability

The intersection of events *A* and *B* (*A* ∩ *B*) is the set of all outcomes in both events *A and B*.



The union of events *A* and *B* (*A* ∪ *B*) is the set of all outcomes in either event *A or B*.

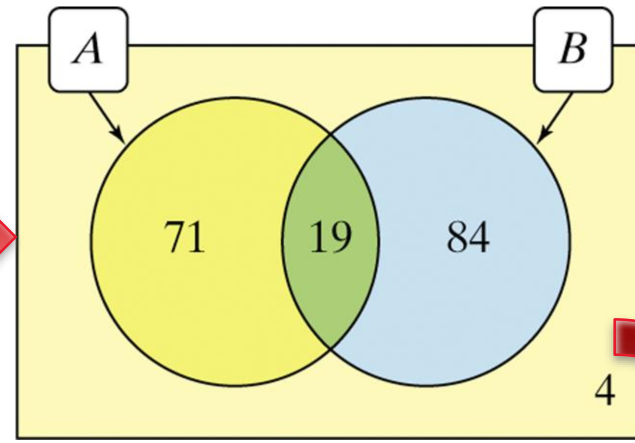


Hint: To keep the symbols straight, remember ∪ for union and ∩ for intersection.

Venn Diagrams and Probability

Recall the example on gender and pierced ears. We can use a Venn diagram to display the information and determine probabilities.

	Pierced Ears?		
Gender	**Yes**	**No**	**Total**
Male	19	71	**90**
Female	84	4	**88**
Total	**103**	**75**	**178**



Probability Rules

Define events *A*: is male and *B*: has pierced ears.

Region in Venn diagram	In words	In symbols	Count
In the intersection of two circles	Male and pierced ears	$A \cap B$	19
Inside circle *A*, outside circle *B*	Male and no pierced ears	$A \cap B^C$	71
Inside circle *B*, outside circle *A*	Female and pierced ears	$A^C \cap B$	84
Outside both circles	Female and no pierced ears	$A^C \cap B^C$	4

Conditional Probability – Car & Home owners

- Consider the population of U.S. adults. If we define events as

Event C: Car owner , and *Event H: Homeowner*

- *Then what is the probability that and adult owns a car or a home?* $P(C \cup H)$

- What is the *probability that and adult owns a car* ***and*** *a home?* $P(H \cap C)$

- *And what do the following conditional probabilities signify?*

$$P(C \mid H) \qquad P(H \mid C) \qquad P(C \mid H^C)$$

Conditional Probability – Car & Home owners

- Given Event C: Car owner , and *Event H: Homeowner, we need more information to solve any compound probabilities or conditional probabilities*

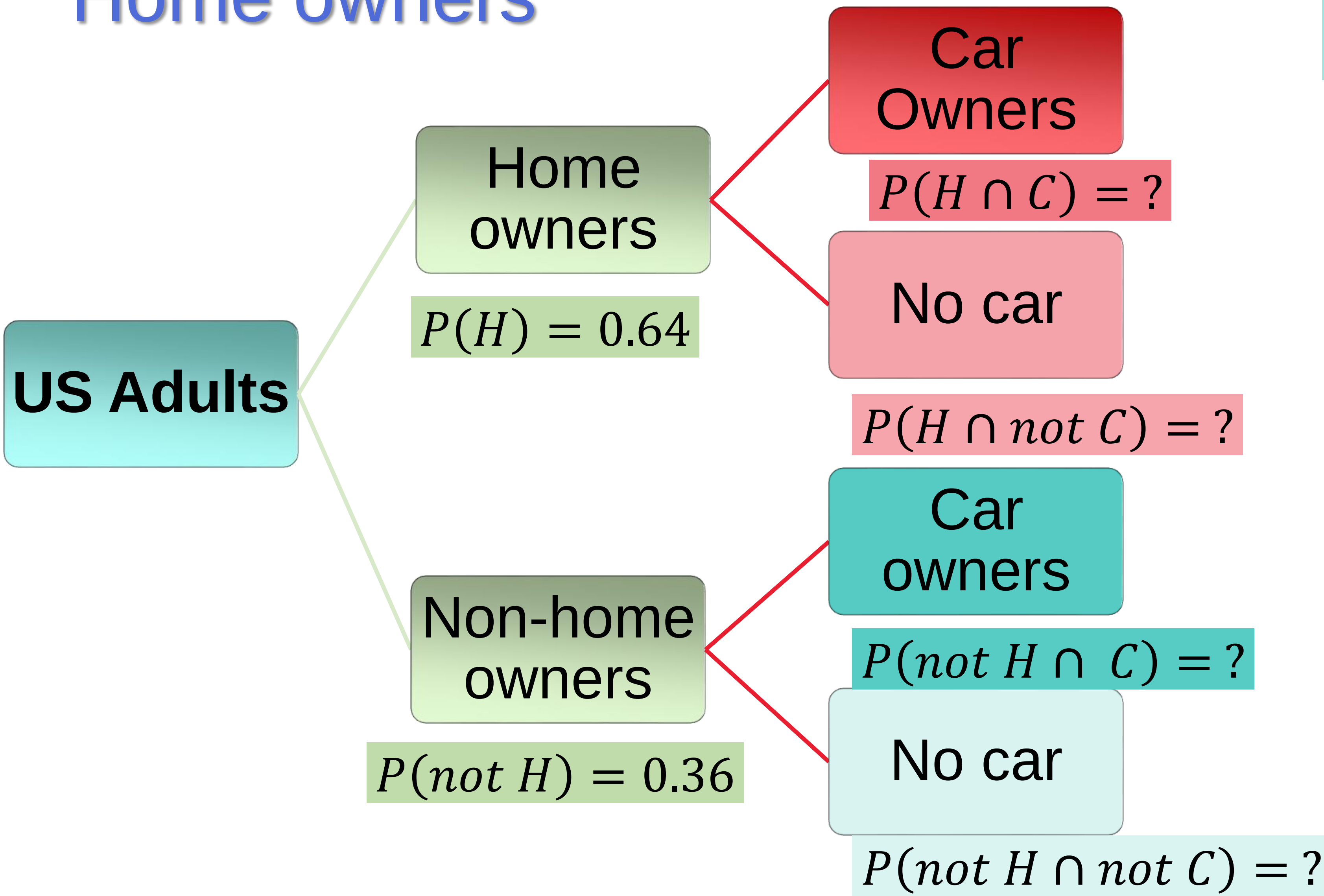
$P(C) = 0.85$ $P(H) = 0.64$

Can we now find the *probability that an adult owns either or both?*

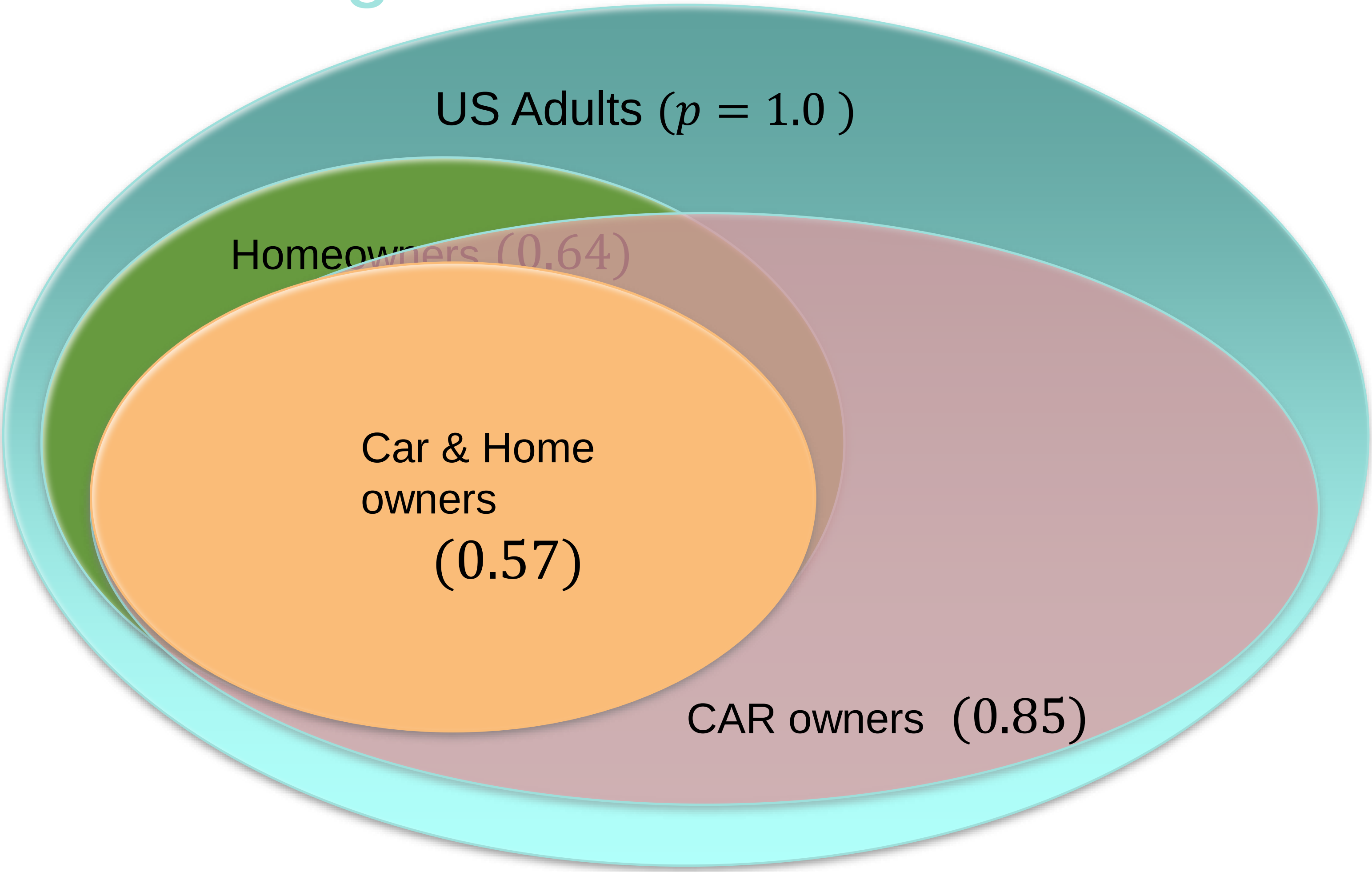
$P(C \cup H)$ $P(H \cap C)$

Are these events independent?
*Are they **mutually exclusive**?*

Conditional Probability – Car & Home owners



Conditional Probability
Venn Diagram



Symbolically $P(H \cap C)$

Conditional Probability

Population: U.S. Adults
Event C: Car owner
Event H: Home owner

P(Car owner)= *P*(**C**) = 0.85
P(Home owner) = *P*(**H**) = 0.64

P(Car owner *or* Homeowner)= $P(\mathrm{C} \cup \mathrm{H}) = P(C) + P(H) - P(H \cap C)$

$\mathrm{P(C|H)} = \mathrm{P}($Car owner given that you own a home)

$$\mathrm{P(C|H)} = \frac{P(\mathrm{C} \cap \mathrm{H})}{P(\mathrm{H})} = \frac{0.57}{0.64} = 0.89$$

$\mathrm{P(H|C)} = P($Home owner given that you own a car)

Section 6.2
Probability Rules

Summary

In this section, we learned that…

- ✓ A **probability model** describes chance behavior by listing the possible outcomes in the **sample space *S*** and giving the probability that each outcome occurs.
- ✓ An **event** is a subset of the possible outcomes in a chance process.
- ✓ For any event A, $0 \le P(A) \le 1$
- ✓ $P(S) = 1$, where S = the sample space
- ✓ If all outcomes in S are equally likely, $P(A) = \dfrac{\text{number of outcomes corresponding to event } A}{\text{total number of outcomes in sample space}}$
- ✓ $P(A^C) = 1 - P(A)$, where A^C is the **complement** of event A; that is, the event that A does not happen.

+

Section 6.2 & 6.3 Probability Rules

Summary

In this section, we learned that…

- ✓ Events *A* and *B* are **mutually exclusive (disjoint)** if they have no outcomes in common. If *A* and *B* are disjoint, $P(A \text{ or } B) = P(A) + P(B)$.
- ✓ A **two-way table** or a **Venn diagram** can be used to display the sample space for a chance process.
- ✓ The **intersection ($A \cap B$)** of events *A* and *B* consists of outcomes in both *A* and *B*.
- ✓ The **union ($A \cup B$)** of events *A* and *B* consists of all outcomes in event *A*, event *B*, or both.
- ✓ The **general addition rule** can be used to find $P(A \text{ or } B)$:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Looking Ahead…

<u>In the next Section…</u>

We'll learn how to calculate conditional probabilities as well as probabilities of independent events.

We'll learn about

- ✓ **Conditional Probability**
- ✓ **Independence**
- ✓ **Tree diagrams and the general multiplication rule**
- ✓ **Special multiplication rule for independent events**
- ✓ **Calculating conditional probabilities**

Extra stuff…

Can you guess what this image shows?

What does this image show?

GUN VIOLENCE Archive 2017

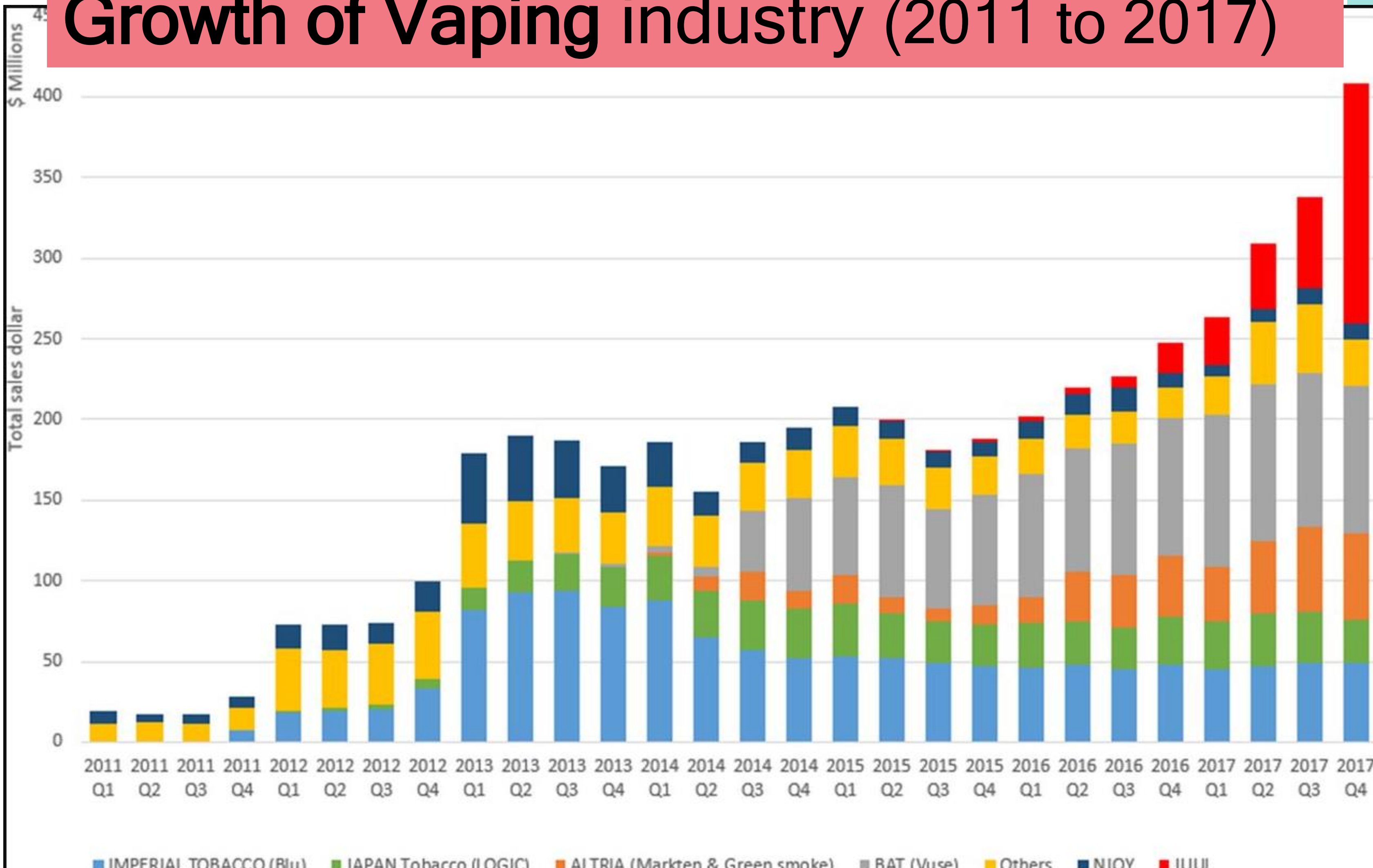
Total Number of Incidents	52,891
Number of Deaths[1]	13,293
Number of Injuries[1]	27,190
Number of Children (age 0-11) Killed or Injured[1]	622
Number of Teens (age 12-17) Killed or Injured[1]	2,793

Source: http://www.gunviolencearchive.org/

The Gun Violence Archive is an online archive of gun violence, incidents collected from over 2,500 media, law enforcement, government and commercial sources daily in an effort to provide near-real time data about the results of gun violence. GVA in an independent data collection and research group with no affiliation with any advocacy organization.

+ What does this image show?

Growth of Vaping industry (2011 to 2017)



Jidong Huang et al. Tob Control 2019;28:146-151

A Randomized Trial of E-Cigarettes versus Nicotine-Replacement Therapy

List of authors.Peter Hajek, Ph.D., Anna Phillips-Waller, B.Sc., Dunja Przulj, Ph.D., et al.

- **BACKGROUND:** E-cigarettes are commonly used in attempts to stop smoking, but evidence is limited regarding their effectiveness as compared with that of nicotine products approved as smoking-cessation treatments.

- **METHODS:** We randomly assigned adults attending U.K. National Health Service stop-smoking services to either nicotine-replacement products of their choice, including product combinations, provided for up to 3 months, or an e-cigarette starter pack (a second-generation refillable e-cigarette with one bottle of nicotine e-liquid [18 mg per milliliter]), with a recommendation to purchase further e-liquids of the flavor and strength of their choice. Treatment included weekly behavioral support for at least 4 weeks. The primary outcome was sustained abstinence for 1 year, which was validated biochemically at the final visit.

A Randomized Trial of E-Cigarettes versus Nicotine-Replacement Therapy

List of authors.Peter Hajek, Ph.D., Anna Phillips-Waller, B.Sc., Dunja Przulj, Ph.D., et al.

- **RESULTS:** A total of 886 participants underwent

Overall, throat or mouth irritation was reported more frequently in the e-cigarette group (65.3%, vs. 51.2% in the nicotine-replacement group) and nausea more frequently in the nicotine-replacement group (37.9%, vs. 31.3% in the e-cigarette group). The e-cigarette group reported greater declines in the incidence of cough and phlegm production from baseline to 52 weeks than did the nicotine-replacement group (relative risk for cough, 0.8; 95% CI, 0.6 to 0.9; relative risk for phlegm, 0.7; 95% CI, 0.6 to 0.9). There were no significant between-group differences in the incidence of wheezing or shortness of breath.

+

A Randomized Trial of E-Cigarettes versus Nicotine-Replacement Therapy

List of authors.Peter Hajek, Ph.D., Anna Phillips-Waller, B.Sc., Dunja Przulj, Ph.D., et al.

Table 5. Respiratory Symptoms at Baseline and at 52 Weeks.*

Symptom	E-Cigarettes (N=315)		Nicotine Replacement (N=279)		Relative Risk (95% CI)†
	Baseline	52 Weeks	Baseline	52 Weeks	
	number (percent)				
Shortness of breath	120 (38.1)	66 (21.0)	92 (33.0)	64 (22.9)	0.9 (0.7–1.1)
Wheezing	102 (32.4)	74 (23.5)	86 (30.8)	59 (21.1)	1.1 (0.8–1.4)
Cough	173 (54.9)	97 (30.8)	144 (51.6)	111 (39.8)	0.8 (0.6–0.9)
Phlegm	137 (43.5)	79 (25.1)	121 (43.4)	103 (36.9)	0.7 (0.6–0.9)

* Symptoms were assessed by asking whether participants had the symptom (yes or no).

† Relative risk was calculated by means of logistic regression. Symptoms at 52 weeks were regressed onto trial group, with adjustment for baseline symptoms and trial center.

A Randomized Trial of E-Cigarettes versus Nicotine-Replacement Therapy

List of authors.Peter Hajek, Ph.D., Anna Phillips-Waller, B.Sc., Dunja Przulj, Ph.D., et al.

Table 2. Abstinence Rates at Different Time Points and Smoking Reduction at 52 Weeks.*

Outcome	E-Cigarettes (N=438)	Nicotine Replacement (N=446)	Primary Analysis: Relative Risk (95% CI)†	Sensitivity Analysis: Adjusted Relative Risk (95% CI)
Primary outcome: abstinence at 52 wk — no. (%)	79 (18.0)	44 (9.9)	1.83 (1.30–2.58)	1.75 (1.24–2.46)‡
Secondary outcomes				
Abstinence between wk 26 and wk 52 — no. (%)	93 (21.2)	53 (11.9)	1.79 (1.32–2.44)	1.82 (1.34–2.47)§
Abstinence at 4 wk after target quit date — no. (%)	192 (43.8)	134 (30.0)	1.45 (1.22–1.74)	1.43 (1.20–1.71)¶
Abstinence at 26 wk after target quit date — no. (%)	155 (35.4)	112 (25.1)	1.40 (1.14–1.72)	1.36 (1.15–1.67)‡
Carbon monoxide–validated reduction in smoking of ≥50% in participants without abstinence between wk 26 and wk 52 — no./total no. (%)	44/345 (12.8)	29/393 (7.4)	1.75 (1.12–2.72)	1.73 (1.11–2.69)‖

* Abstinence at 52 weeks was defined as a self-report of smoking no more than five cigarettes from 2 weeks after the target quit date, validated biochemically by an expired carbon monoxide level of less than 8 ppm at 52 weeks. Abstinence between week 26 and week 52 was defined as a self-report of smoking no more than five cigarettes between week 26 and week 52, plus an expired carbon monoxide level of less than 8 ppm at 52 weeks. Abstinence at 4 weeks was defined as a self-report of no smoking from 2 weeks after the target quit date, plus an expired carbon monoxide level of less than 8 ppm at 4 weeks. Abstinence at 26 weeks was defined as a self-report of smoking no more than five cigarettes from 2 weeks after the target quit date to 26 weeks; there was no validation by expired carbon monoxide level.