

+ Today's AGENDA...

In today's lesson...

We'll learn how to display quantitative data.

- ✓ **Review Dotplots**
- ✓ **Introduce Stemplots**
- ✓ **Introduce Histograms**

We'll also learn how to describe and compare distributions of quantitative data.



Today's Lesson

Displaying Quantitative Data with Graphs



Learning Objectives

After this section, you should be able to...

- ✓ CONSTRUCT and INTERPRET dotplots, stemplots, and histograms
- ✓ DESCRIBE the shape of a distribution
- ✓ COMPARE distributions
- ✓ USE histograms wisely

+ Warm-UP

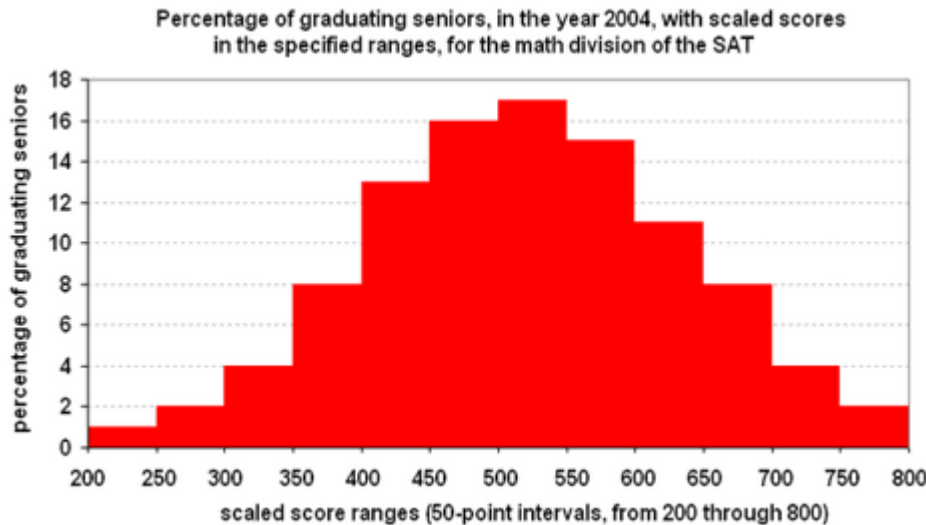
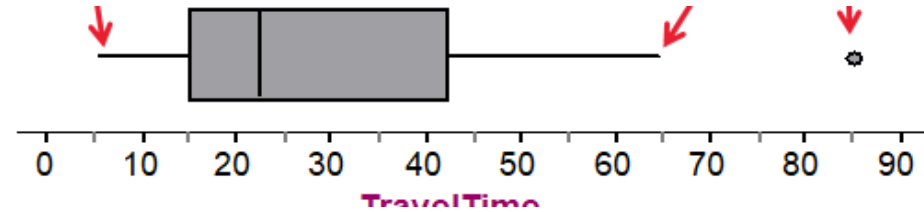
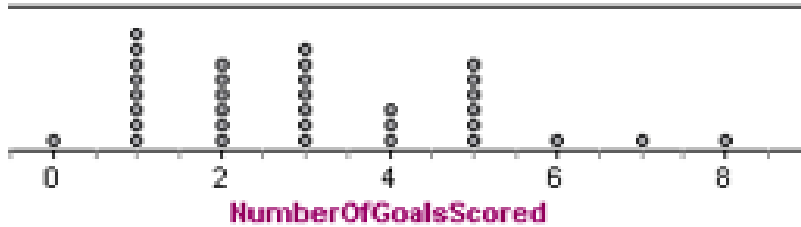
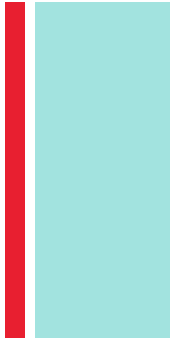
- 1) What is the acronym we use to remember describing the different features or attributes of quantitative distributions?
- 2) What are the different types of graphical or visual charts that we use for quantitative data?
- 3) What is the five number summary?

Definition:

The **five-number summary** of a distribution consists of the smallest observation, the first quartile, the median, the third quartile, and the largest observation, written in order from smallest to largest.

Min Q_1 *M* Q_3 *Max*

+ Graphical Displays of numerical data



1	33359
2	233466
3	0148
4	9
5	0017

Order leaves

Examining the Distribution of a Quantitative Variable

- The purpose of a graph is to help us understand the data. After you make a graph, always ask, “What do I see?”

How to Examine the Distribution of a Quantitative Variable

In any graph, look for the **overall pattern** and for striking **departures** from that pattern.

Describe the overall pattern of a distribution by its:

- **Shape**
- **Center**
- **Spread**

Don't forget your
SOCS!

Note individual values that fall outside the overall pattern. These departures are called **outliers**.

■ Describing Shape

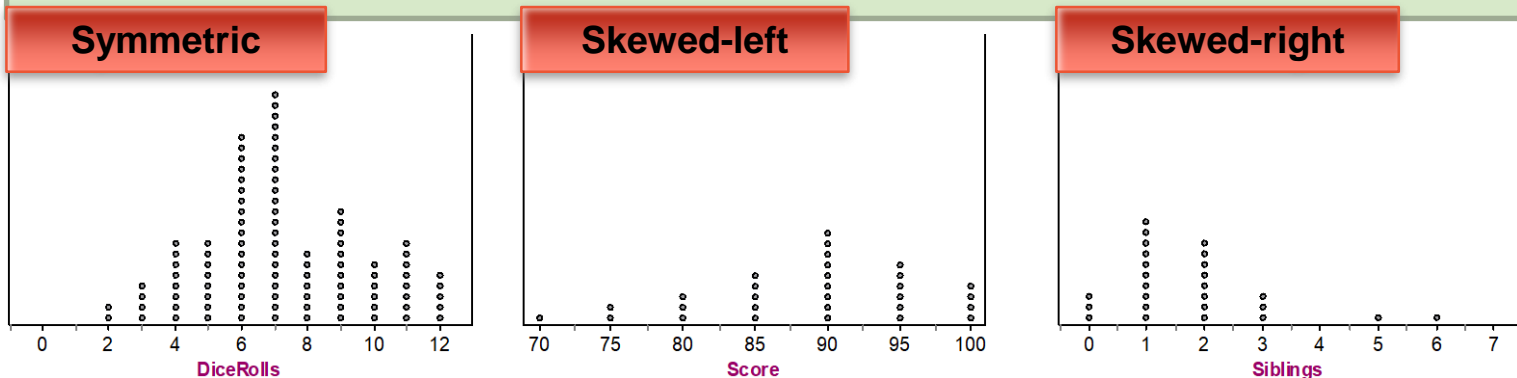
- When you describe a distribution's shape, concentrate on the main features. Look for rough **symmetry** or clear **skewness**.

Definitions:

A distribution is roughly **symmetric** if the right and left sides of the graph are approximately mirror images of each other.

A distribution is **skewed to the right** (right-skewed or *positively skewed*) if the right side of the graph (containing the half of the observations with larger values) is much longer than the left side.

It is **skewed to the left** (left-skewed or *negatively skewed*) if the left side of the graph is much longer than the right side.



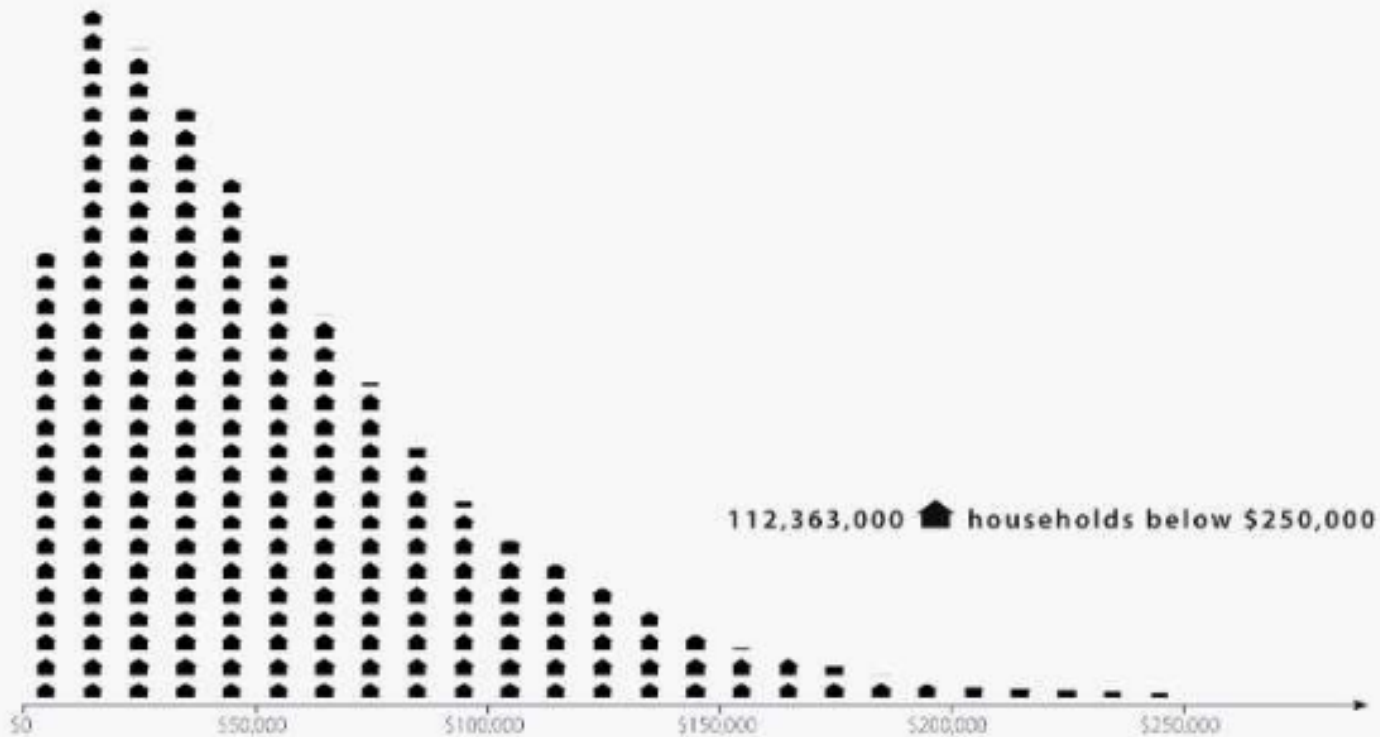


Visualizing Economics
Making the Invisible Hand Visible

Visit www.visualizingeconomics.com
to view more examples

2005 United States
Income Distribution (Bottom 98%)
Each 🏠 equals 500,000 households

U.S. Income Distribution from 2005



Skewed Right or *positively skewed* →

+
Distribution Shape

■ Review of Dotplots

- One of the simplest graphs to construct and interpret is a **dotplot**. Each data value is shown as a dot above its location on a number line.

How to Make a Dotplot

- 1) Draw a horizontal axis (a number line) and label it with the variable name.
- 2) Scale the axis from the minimum to the maximum value.
- 3) Mark a dot above the location on the horizontal axis corresponding to each data value.

Number of Goals Scored Per Game by the 2004 US Women's Soccer Team

3	0	2	7	8	2	4	3	5	1	1	4	5	3	1	1	3
3	3	2	1	2	2	2	4	3	5	6	1	5	5	1	1	5



■ Measuring Center: The Mean

- The most common measure of center is the ordinary arithmetic average, or **mean**.

Definition:

To find the **mean** \bar{x} (pronounced “x-bar”) of a set of observations, add their values and divide by the number of observations. If the n observations are $x_1, x_2, x_3, \dots, x_n$, their mean is:

$$\bar{x} = \frac{\text{sum of observations}}{n} = \frac{x_1 + x_2 + \dots + x_n}{n}$$

In mathematics, the capital Greek letter Σ (Sigma) is short for “add them all up.” Therefore, the formula for the mean can be written in more compact notation:

$$\bar{x} = \frac{\sum x_i}{n}$$

■ Measuring Center: The Median

- Another common measure of center is the **median**. In section 1.2, we learned that the median describes the midpoint of a distribution.

Definition:

The **median M** is the midpoint of a distribution, the number such that half of the observations are smaller and the other half are larger.

To find the median of a distribution:

- 1) Arrange all observations from smallest to largest.
- 2) If the number of observations **n is odd**, the median M is the center observation in the ordered list.
- 3) If the number of observations **n is even**, the median M is the *average of the two center observations* in the ordered list.

Find the mean of this mean & median of this data:

MODEL	MPG	MODEL	MPG	MODEL	MPG
Acura RL	22	Dodge Avenger	30	Mercedes-Benz E350	24
Audi A6 Quattro	23	Hyundai Elantra	33	Mercury Milan	29
Bentley Arnage	14	Jaguar XF	25	Mitsubishi Galant	27
BMW 5281	28	Kia Optima	32	Nissan Maxima	26
Buick Lacrosse	28	Lexus GS 350	26	Rolls Royce Phantom	18
Cadillac CTS	25	Lincoln MKZ	28	Saturn Aura	33
Chevrolet Malibu	33	Mazda 6	29	Toyota Camry	31
Chrysler Sebring	30	Mercedes-Benz E350	24	Volkswagen Passat	29

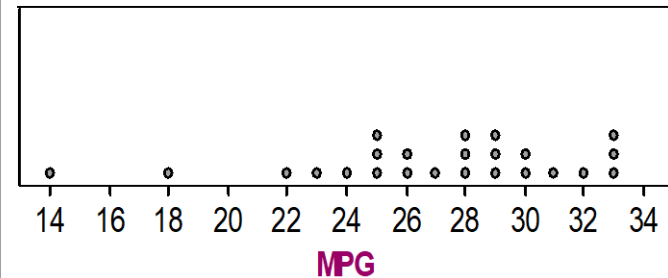
CENTERS (measures of central tendency): *mean, median, mode*

CENTER: *Mean = 26.958 mpg,*
Median = 28 mpg, Mode = multimodal

■ **Examine this data** Example

- The table and dotplot below displays the Environmental Protection Agency's estimates of highway gas mileage in miles per gallon (MPG) for a sample of 24 model year 2009 midsize cars.

MODEL	MPG	MODEL	MPG	MODEL	MPG
Acura RL	22	Dodge Avenger	30	Mercedes-Benz E350	24
Audi A6 Quattro	23	Hyundai Elantra	33	Mercury Milan	29
Bentley Arnage	14	Jaguar XF	25	Mitsubishi Galant	27
BMW 5281	28	Kia Optima	32	Nissan Maxima	26
Buick Lacrosse	28	Lexus GS 350	26	Rolls Royce Phantom	18
Cadillac CTS	25	Lincoln MKZ	28	Saturn Aura	33
Chevrolet Malibu	33	Mazda 6	29	Toyota Camry	31
Chrysler Sebring	30	Mercedes-Benz E350	24	Volkswagen Passat	29



Describe the shape, center, and spread of the distribution. Are there any outliers?

SHAPE: *skewed left or neg. skewed*

CENTER: *mean, Median, mode?*

SPREAD: *range is 19 (33 – 14)*

Comparing the Mean and the Median

- The mean and median measure center in different ways, and both are useful.
 - *Don't confuse the "average" value of a variable (the mean) with its "typical" value, which we might describe by the median.*

Comparing the Mean and the Median

The mean and median of a roughly symmetric distribution are close together.

If the distribution is exactly symmetric, the mean and median are exactly the same.

In a skewed distribution, the mean is usually farther out in the long tail than is the median.

■ Measuring Center

- Use the data below to calculate the mean and median of the commuting times (in minutes) of 20 randomly selected New York workers.

Example

10	30	5	25	40	20	10	15	30	20	15	20	85	15	65	15	60	60	40	45
----	----	---	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----

$$\bar{x} = \frac{10 + 30 + 5 + 25 + \dots + 40 + 45}{20} = 31.25 \text{ minutes}$$

0	5
1	005555
2	0005
3	00
4	005
5	
6	005
7	
8	5

Key: 4|5
represents a
New York
worker who
reported a 45-
minute travel
time to work.

$$M = \frac{20 + 25}{2} = 22.5 \text{ minutes}$$

■ Stemplots (Stem-and-Leaf Plots)

- Another simple graphical display for small data sets is a stemplot. Stemplots give us a quick picture of the distribution while including the actual numerical values.

How to Make a Stemplot

- 1) Separate each observation into a **stem** (all but the final digit) and a **leaf** (the final digit).
- 2) Write all possible stems from the smallest to the largest in a vertical column and draw a vertical line to the right of the column.
- 3) Write each leaf in the row to the right of its stem.
- 4) Arrange the leaves in increasing order out from the stem.
- 5) Provide a key that explains in context what the stems and leaves represent.

■ Stemplots (Stem-and-Leaf Plots)

Question:
How many pairs of shoes do you have?

1	
2	
3	
4	
5	

Stems

1		93335
2		664233
3		1840
4		9
5		0701

Add leaves

1		33359
2		233466
3		0148
4		9
5		0017

Order leaves

Key: 4|9
represents a
female student
who reported
having 49
pairs of shoes.

Add a key

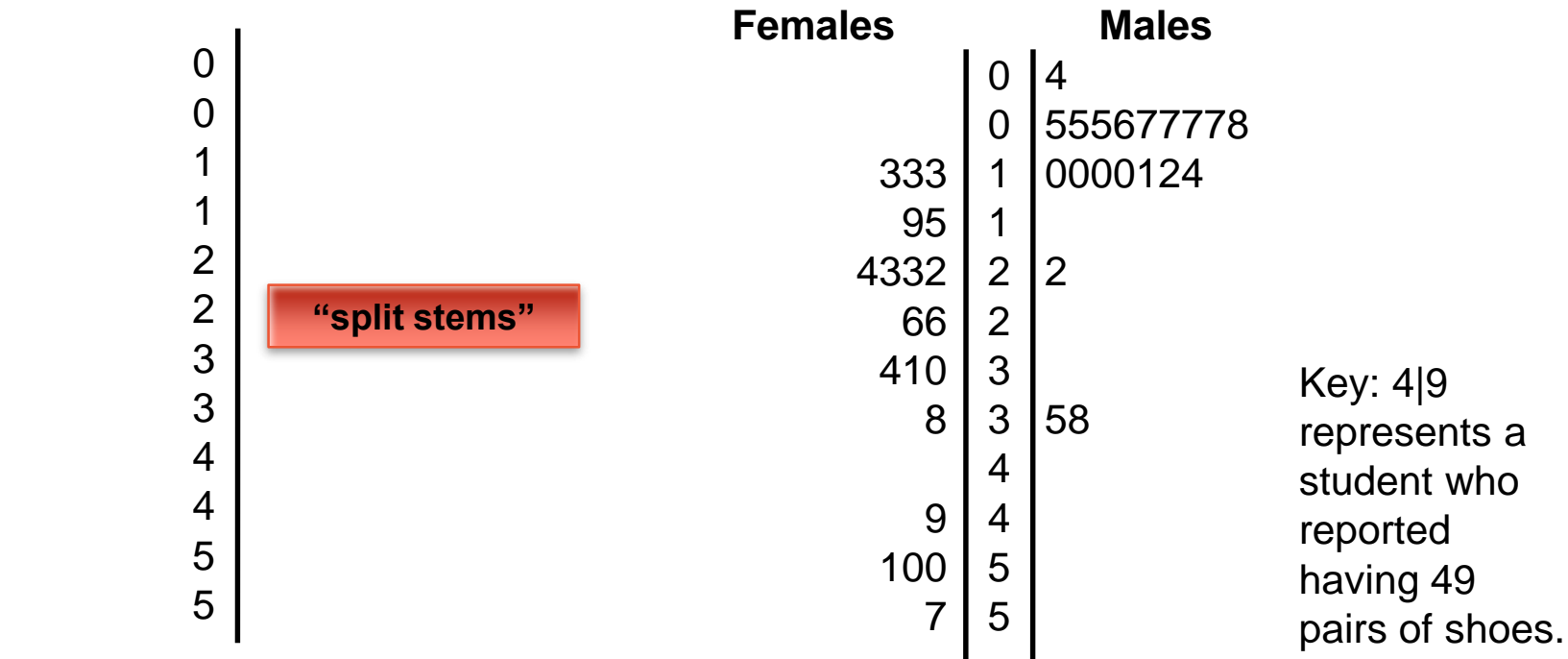
■ Splitting Stems and Back-to-Back Stemplots

- When data values are “bunched up”, we can get a better picture of the distribution by **splitting stems**.
- Two distributions of the same quantitative variable can be compared using a **back-to-back stemplot** with common stems.

Females

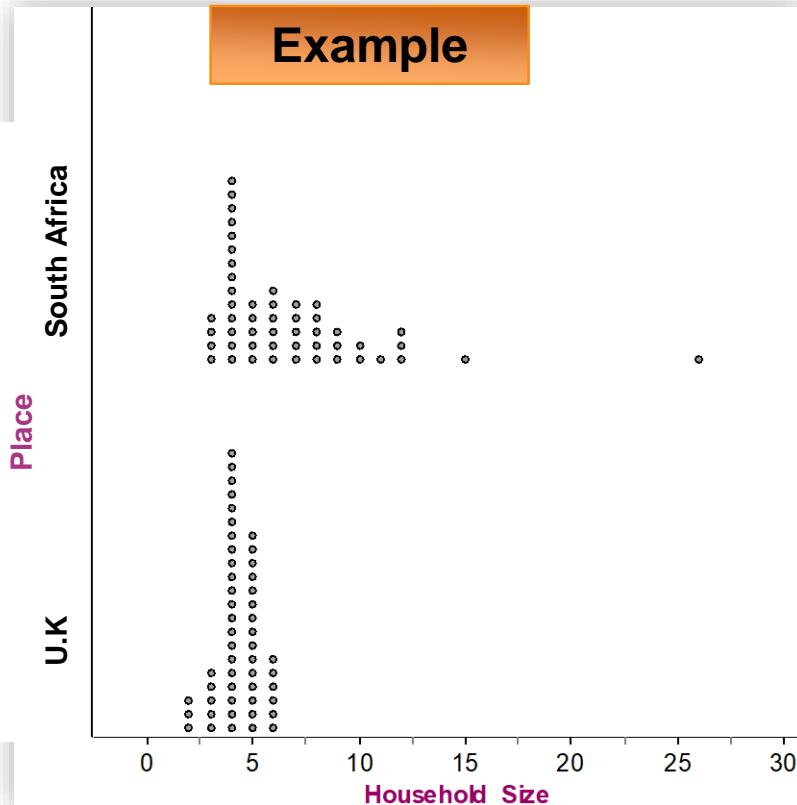
Males

50	26	26	31	57	19	24	22	23	38	14	7	6	5	12	38	8	7	10	10
13	50	13	34	23	30	49	13	15	51	10	11	4	5	22	7	5	10	35	7



■ Comparing Distributions

- Some of the most interesting statistics questions involve comparing two or more groups.
- Always discuss **shape**, **center**, **spread**, and possible **outliers** whenever you compare distributions of a quantitative variable.



Compare the distributions of household size for these two countries. Don't forget your **SOCS**!

+ Helicopter Design

- You have 3 minutes to design a helicopter using a ½ sheet of paper
- You may **not** use any **addition**
- You are allowed to use **subtraction**
- **Coloring** is optional
- Design augments/changes are welcomed after testing

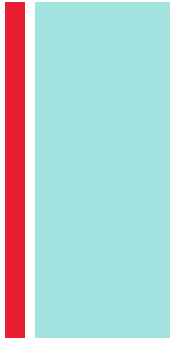
Desired characteristics (response variables)

1. Rotation

2. Vertical descent (*or a decent vertical descent* 😊)



AGENDA – Oct 25 2022



- Put your name on Test review (HW #12) and pass to front
- Warm-UP
- Video: Histograms
- Review & practice box plots & histograms
- Test review answers
- **TEST #2** on Thursday



October 12, 2022 Warm UP

Practicing Box and Whisker Plots with the 5 Number summary (*already completed?*)

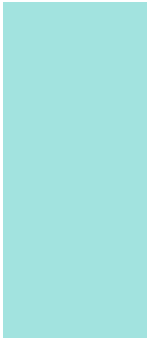
1. Find the five number summary for each set of data:

Data Set A: 4, 5, 7, 9, 11

Data Set B: 4, 5, 7, 8, 9, 11

2. Find the IQR for each set

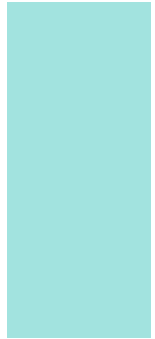
3. Make a box plot for each data set.





Warm UP

Practicing Box and Whisker Plots with the 5 Number summary



1. Five number summary:

Data Set A: 4, 5, 7, 9, 11

Min	Q_1	Med	Q_3	Max
4	4.5	7	10	11

Data Set B: 4, 5, 7, 8, 9, 11

Min	Q_1	Med	Q_3	Max
4	5	7.5	9	11

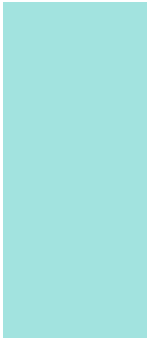
$$\frac{\text{IQR}}{(Q_3 - Q_1)}$$

$$\text{IQR} = 5.5$$

$$\text{IQR} = 4$$



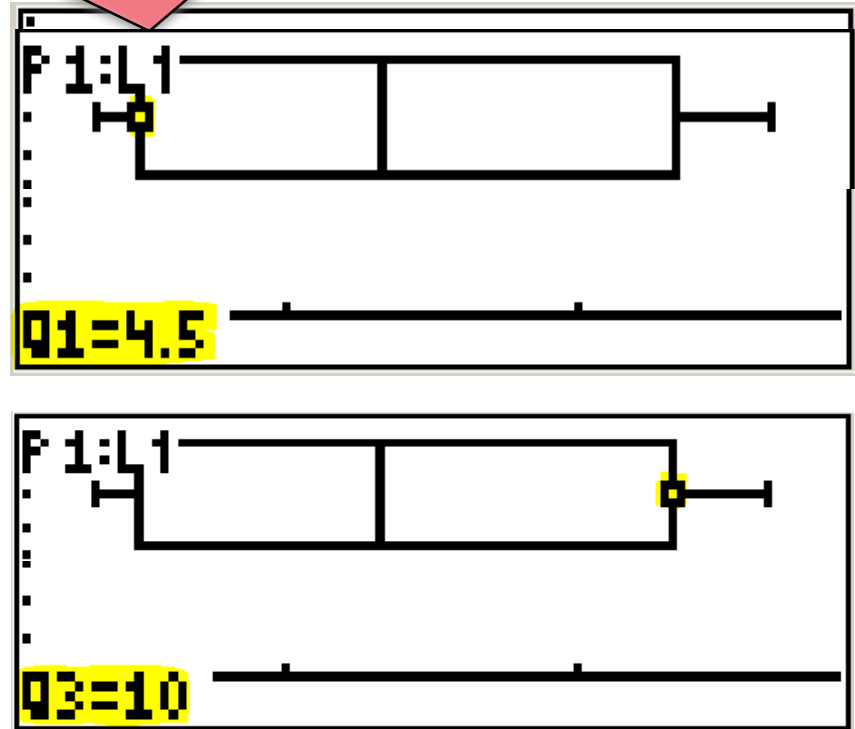
Comparing Box Plots & Five Number summary



Sample Data

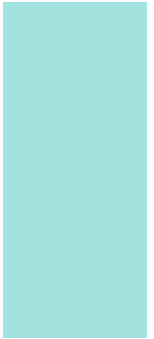
Sample Box and Whisker Plots

L1	L2
5 5 6 8 11	5 5 6 8 11





Review Frequency Distributions & Bar Charts for Categorical Data



- **Frequency Distribution:** A table that displays the possible categories along with the associated *frequencies* (the count or number of times it occurs)
- **Relative Frequency Distribution:** A table that displays the possible categories along with the proportion of observations for each category.
- $relative\ frequency = \frac{frequency}{total\ observations\ in\ data\ set}$

- **Categorical Variables** place individuals into one of several groups or categories
 - The values of a categorical variable are labels for the different categories
 - The distribution of a categorical variable lists the count or percent of individuals who fall into each category.

Example

Frequency Table	
Format	Count of Stations
Adult Contemporary	1556
Adult Standards	1196
Contemporary Hit	569
Country	2066
News/Talk	2179
Oldies	1060
Religious	2014
Rock	869
Spanish Language	750
Other Formats	1579
Total	13838

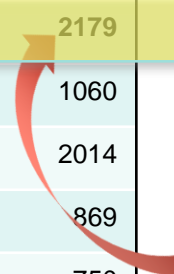
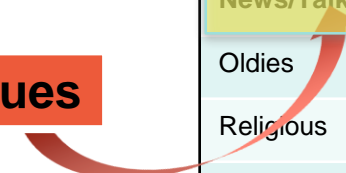
Relative Frequency Table	
Format	Percent of Stations
Adult Contemporary	11.2
Adult Standards	8.6
Contemporary Hit	4.1
Country	14.9
News/Talk	15.7
Oldies	7.7
Religious	14.6
Rock	6.3
Spanish Language	5.4
Other Form	11.4
Total	99.9

Variable

Values

Count

Percent



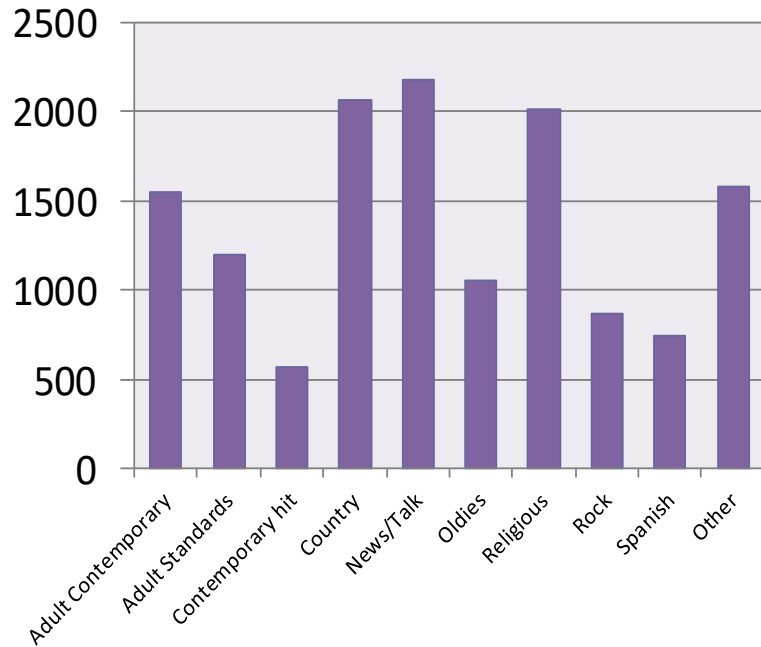
Displaying categorical data

Frequency tables can be difficult to read.

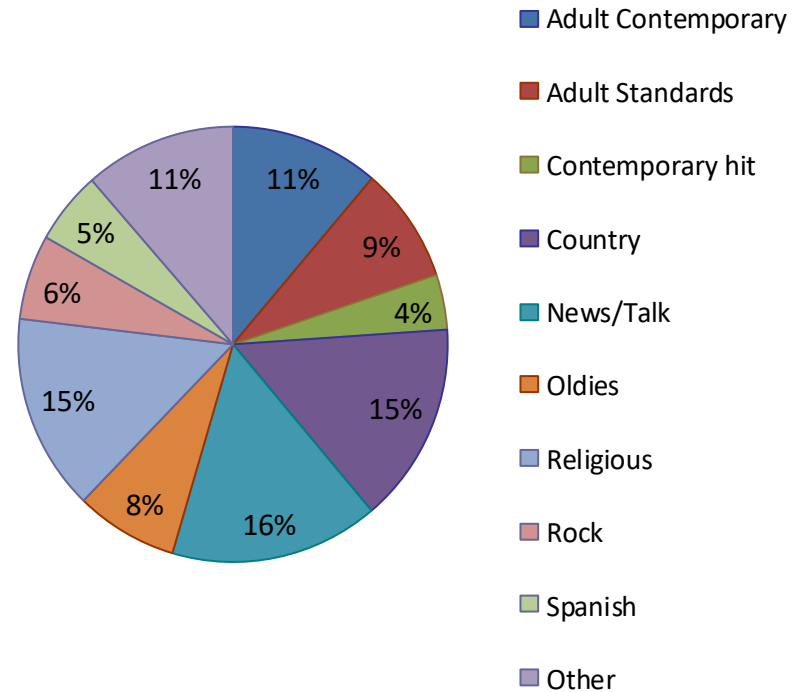
Sometimes it is easier to analyze a distribution by displaying it with a **bar graph** or **pie chart**.

Relative Frequency Table

Count of Stations



Percent of Stations

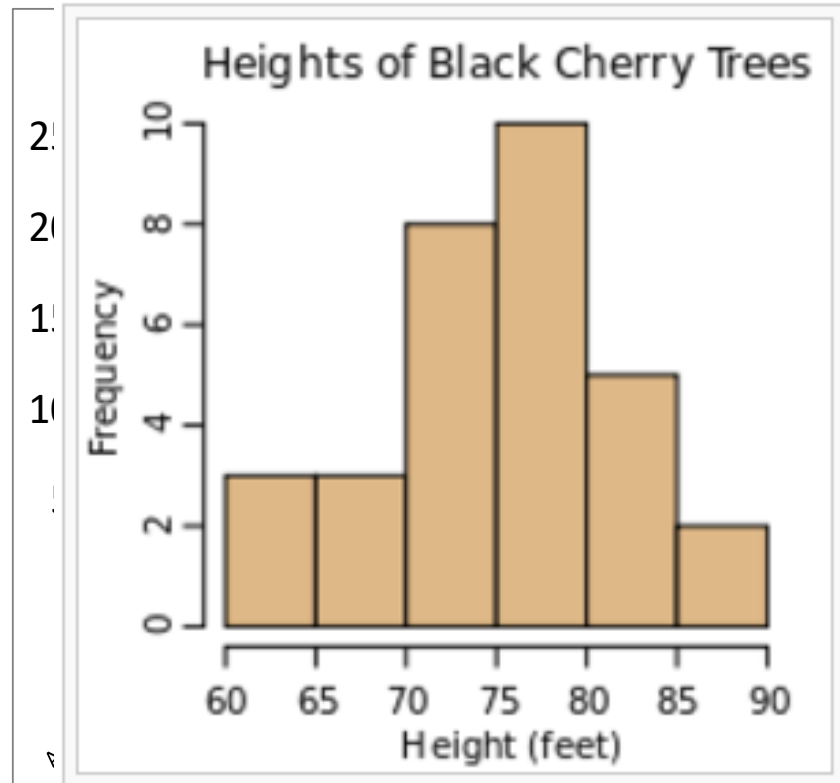


■ Displaying categorical data

Frequency tables can be easier to analyze by displaying the distribution with a **bar graph**.

Compare these 2 graphical displays:

Frequency Table	
Format	Count of Stations
Adult Contemporary	1556
Adult Standards	1196
Contemporary Hit	569
Country	2066
News/Talk	2179
Oldies	1060
Religious	2014
Rock	869
Spanish Language	750
Other Formats	1579
Total	13838



Bar Graphs vs. Histograms (or Bar Charts)

Bar charts and histograms compare sizes of different groups.

Bar charts

- Qualitative groups
- Symmetry and skewness not used
- Space between columns
- Columns can be vertical or horizontal

Histograms

- Quantitative groups
- Symmetry and skewness are used
- No space between columns
- Columns are always vertical

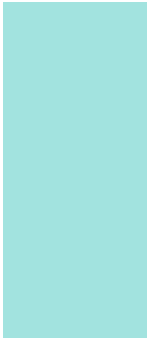
Bar Graphs \neq Histograms

Bar Graphs (or Charts)

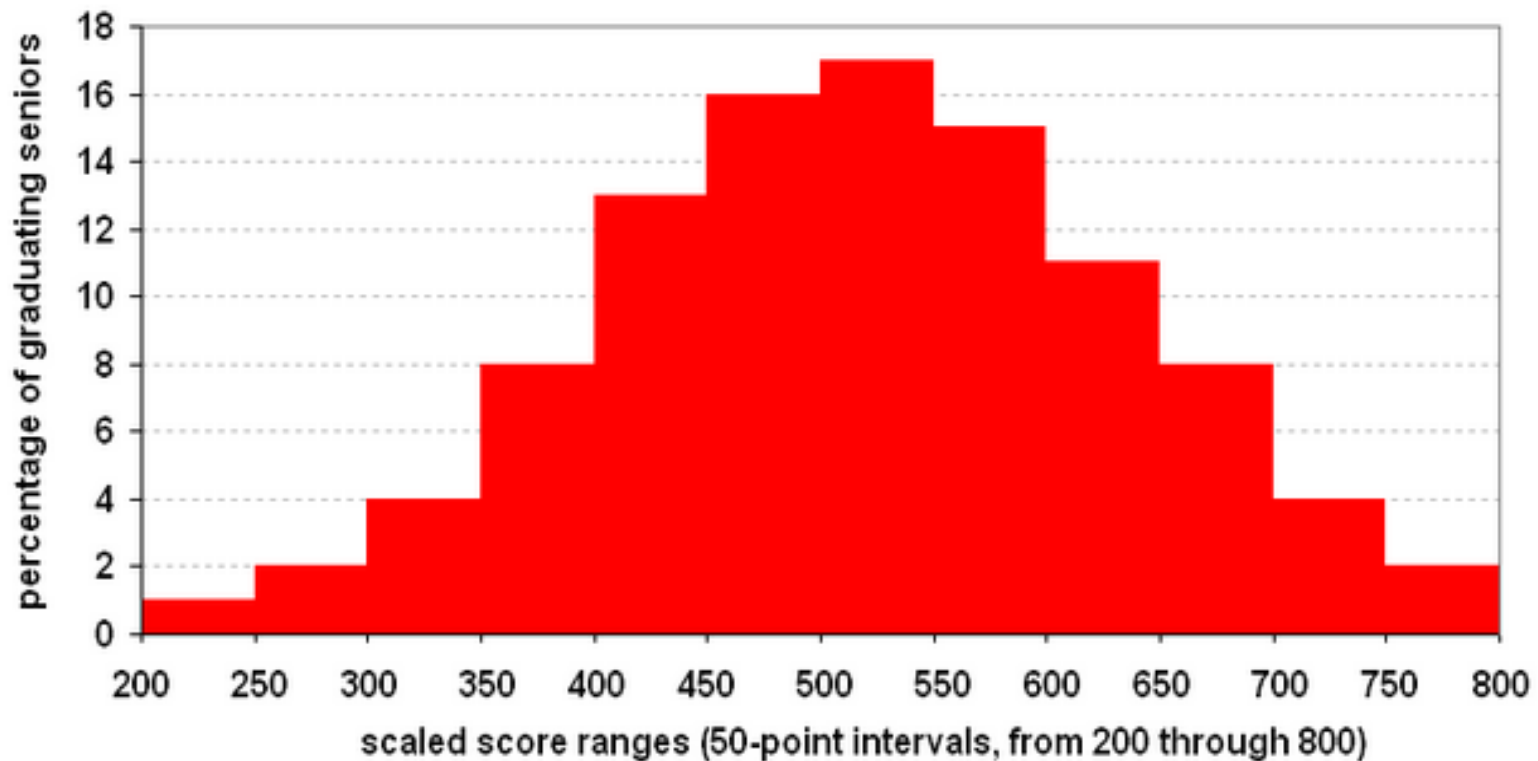
Video Link: <http://stattrek.com/statistics/charts/histogram.aspx?Tutorial=AP>



Displays of Numerical Data: Frequency Distributions using Histograms



Percentage of graduating seniors, in the year 2004, with scaled scores in the specified ranges, for the math division of the SAT



■ Histograms

- Quantitative variables often take many values. A graph of the distribution may be clearer if nearby values are grouped together.
- The most common graph of the distribution of one quantitative variable is a **histogram**.

How to Make a Histogram

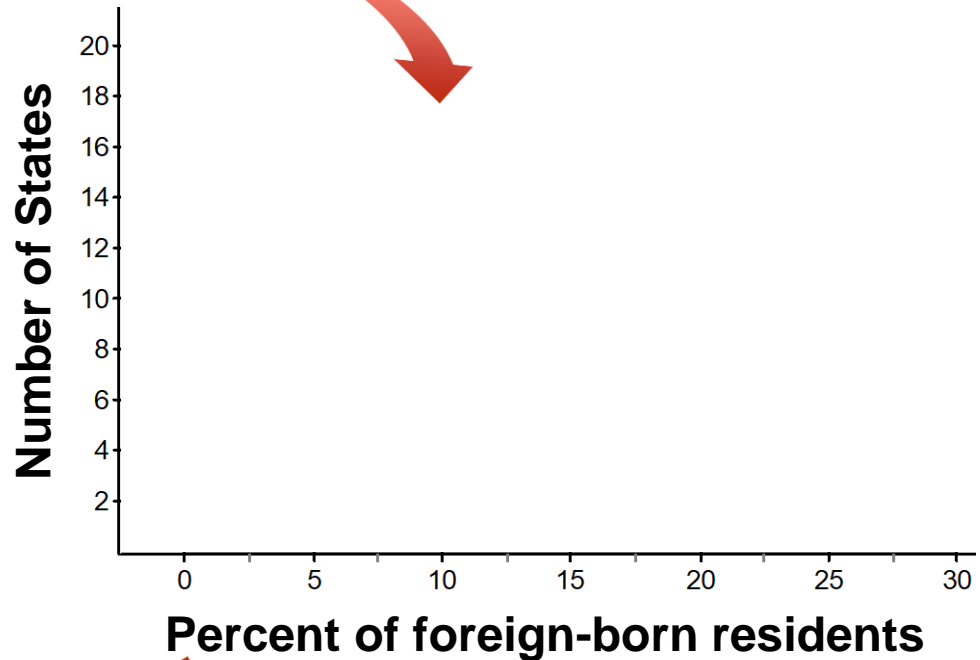
- 1) Divide the range of data into classes of equal width.
- 2) Find the count (*frequency*) or percent (*relative frequency*) of individuals in each class.
- 3) Label and scale your axes and draw the histogram. The height of the bar equals its frequency. Adjacent bars should touch, *unless* a class contains no individuals (observations).

Example

■ Making a Histogram

- The table below presents data on the percent of residents from each state who were born outside of the U.S.

Class	Count
0 to <5	20
5 to <10	13
10 to <15	9
15 to <20	5
20 to <25	2
25 to <30	1
Total	50



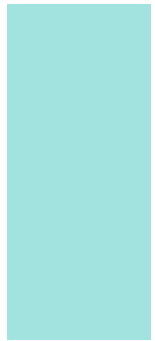


Practice Example: Percent of OLD People

- Use your TI-84 calculator to input the data regarding Percent of residents aged 65 or older in each state.
- From this data, we want to generate a histogram to graphically represent the data.

```
2011 CALC TESTS
1: Edit...
2: SortA(
3: SortD(
4: ClrList
5: SetUpEditor
```

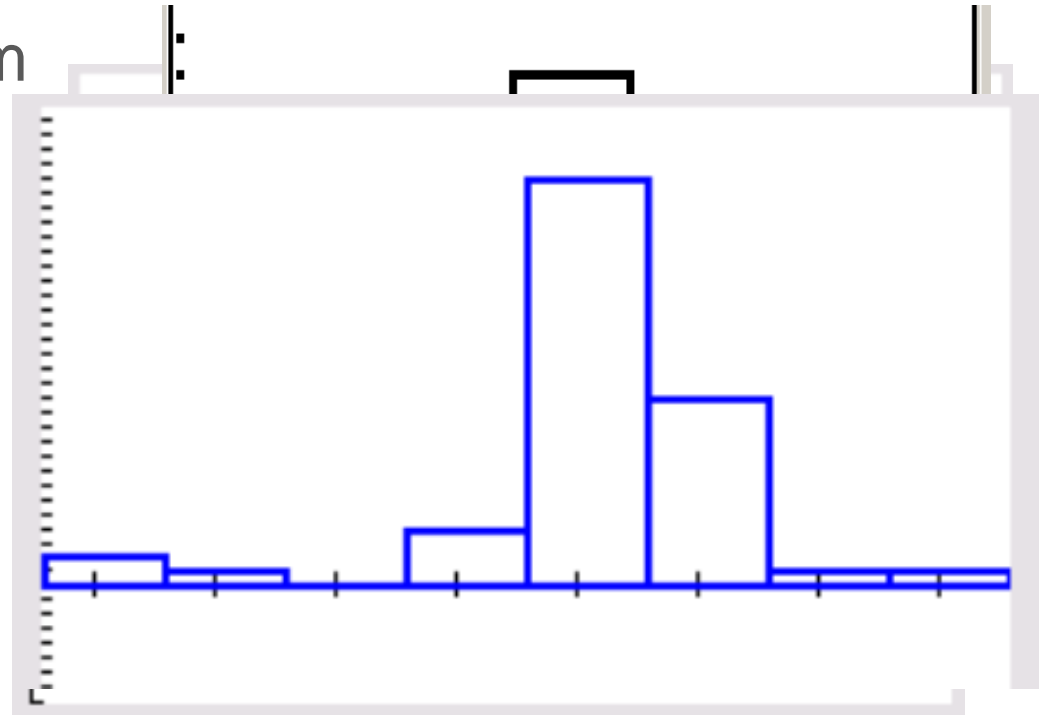
L1	L2	L3	2
10	AP	-----	
30	2		
5	5		
25	6		
...			





Practice Example: Percent of residents ≥ 65 years old

- Generate a Histogram with you TI-84 calculators
- What is your window setting?
- Can you change the intervals?
- Questions



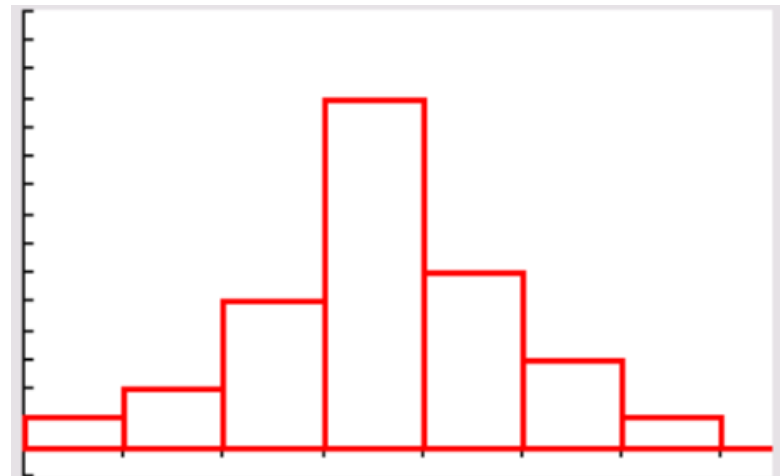
+ Histograms on TI-84

- You can change the **zoom**:

```
ZOOM MEMORY
1:ZBox
2:Zoom In
3:Zoom Out
4:ZDecimal
5:ZSquare
6:ZStandard
7:ZTrig
8:ZInteger
9↓ZoomStat
```

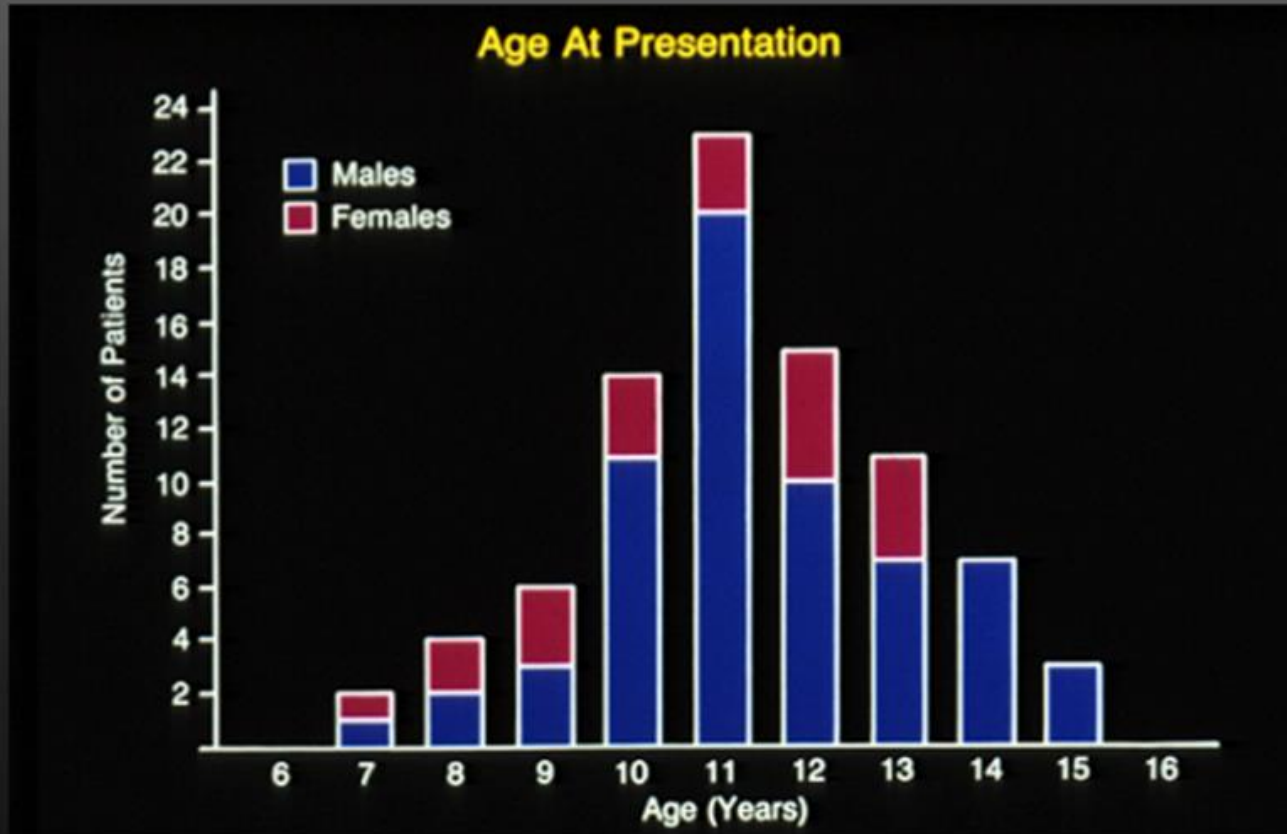
- You can change the **intervals**:

```
WINDOW
Xmin=0
Xmax=13
Xscl=?
Ymin=-1
Ymax=10
Yscl=1
```





Bar Chart or Histogram?



Prevention and Management of Calcaneal Apophysitis Children: An Overuse Syndrome.
Lyle J. Micheli, M.D., and M. Lloyd Ireland, M.D.

Journal of Pediatric Orthopedics 7:34 - 38 © 1987 Raven Press, New York

■ Using Histograms Wisely

- Here are several cautions based on common mistakes students make when using histograms.

Cautions

- 1) Don't confuse *histograms* and *bar graphs*.
- 2) When comparing distributions with different numbers of observations (different size samples) use *percents* instead of counts on the vertical axis (*relative frequency*).
- 3) Choose the best graphical display: *bar chart*, *dot plot*, or *histogram*. Just because a graph looks nice, it's not necessarily a meaningful display of data.