Chapter 6: Probability Test REVIEW - ANSWERS

1. How is the the conditional probability that the event E occurs given that event F has already occurred different from the *P (event F)* given that E has already occurred?

   Conditional probability is calculated when you "divide by the given"

   So $P(E\,|F) = \frac{P(E\cap F)}{P(F)}$ but $P(F\,|E) = \frac{P(E\cap F)}{P(E)}$

2. If P(A) = 0.5, P(B) = 0.3, and P(A∩B) = 0.15, then:
   a. A and B are disjoint events
   b. A and B are dependent events
   c. A and B are independent events
   d. A and B are not disjoint
   e. A and B are neither disjoint nor independent

3. If a fair coin is flipped three times with the outcome of each flip independent of each other, then the probability that at least one of the three flips results in a head is what?

   Ans: $\boldsymbol{P(at\ least\ 1\ head) = P(no\ heads)^C = 1 - (0.5)^3 = 0.875}$

4. Only 20% of the applicants for new positions at a large software company are female. Assuming that two positions will be filled independently of each other, what is the probability that both positions are filled by females?

   Ans: $\boldsymbol{.2 \times .2 = 0.04}$

5. A family is going shopping for a new van. The probability that the family will purchase a Ford van is 0.33, a Chevy van is 0.25, a Dodge van 0.20, and a Toyota van 0.22. The probability that the family purchases a Toyota van or a Ford van or a Chevy van is:
   a. $.33 \times .22 \times .25$
   b. $1 - .33 \times .22 \times .25$
   c. $.33 + .22 + .25$
   d. $1 - (.33 + .22 + .25)$
   e. None of the above

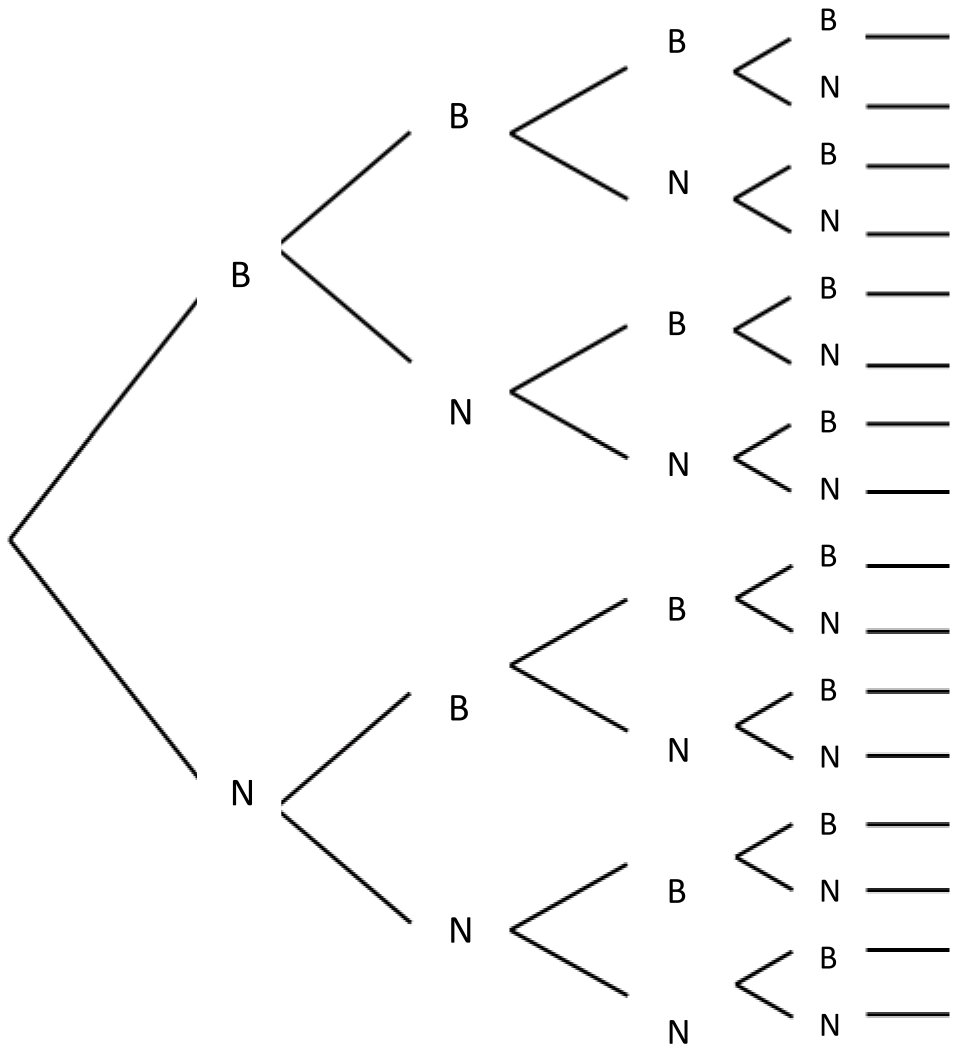
6. Suppose there are 60 students in a statistics class of which 24 are female. If three students are selected without replacement to work problems at the board, what is the probability that all three of the students chosen are female?

   Ans: $\boldsymbol{\frac{24}{60} \times \frac{23}{59} \times \frac{22}{58} \approx 0.0591}$ **or approx. 5.9 %**

7. The probability that a new tire will have a blowout in the first year is 0.10. If the four tires on a new car function independently of each other, what is the probability that at least one tire blows out in the first year?

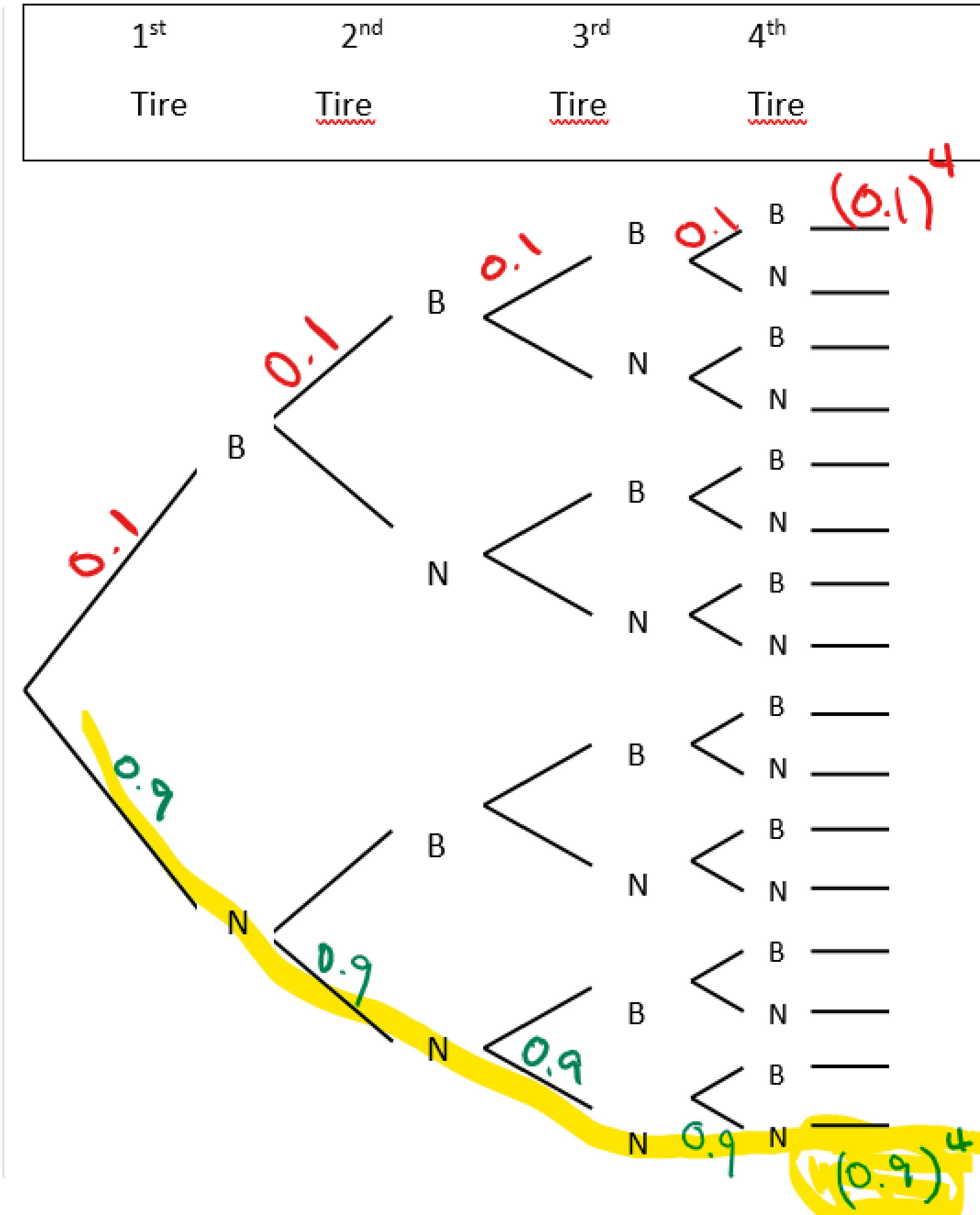
   a. $.10 \times .10 \times .10 \times .10$
   b. $1 - (.10 \times .10 \times .10 \times .10)$
   c. $.90 \times .90 \times .90 \times .90$
   d. $1 - (.90 \times .90 \times .90 \times .90)$
   e. None of the above

1st	2nd	3rd	4th
Tire	Tire	Tire	Tire



Tire has a Blowout = B

No blowout = N



Tire has a Blowout = B 0.1

No blowout = N = 0.9

P(at least one tire has blowout) = 1 – P(NO tires have blowout)

8. A complex electronic device contains three components, A, B, and C. The probabilities of failure for each component in any one year are 0.01, 0.03, and 0.04, respectively. If any one component fails, the device will fail. Assuming the components fail independently of one another, what is the probability that the device will **not** fail in one year?

A. Less than 0.01
B. 0.078
C. 0.080
D. 0.922
E. Greater than 0.99

P(Success part A) = 0.99	$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$
P(Success part B) = 0.97	$P(A \cap B \cap C) = 0.99 \cdot 0.97 \cdot 0.96$
P(Success part C) = 0.96	

9. A large store has a customer service department where customers can go to ask for help with store-related issues. According to store records, approximately twenty-five percent of all customers who go to the service department ask for help finding an item. Assume the reason each customer goes to the service department is independent from customer to customer. Based on the approximation, what is the probability that at least 1 of the next 4 customers who go to the service department will ask for help finding an item?

A. $4\left(\frac{1}{4}\right)^2$

B. $1-\left(\frac{1}{4}\right)^2$

C. $1-\left(\frac{1}{4}\right)^4$

D. $1-\left(\frac{3}{4}\right)^4$

E. $\left(\frac{4}{4}\right)\left(\frac{3}{4}\right)\left(\frac{2}{4}\right)\left(\frac{1}{4}\right)$

P(at least one asks HELP) = $P(one\ asks) + P(two\ ask) + P(3\ ask) + P(4\ ask)$

OR Complement RULE: $P(A) = 1 - P(A^C)$ and $P(A^C) = 1 - P(A)$

P(at least one asks HELP) = 1 – P(NO one asks HELP)

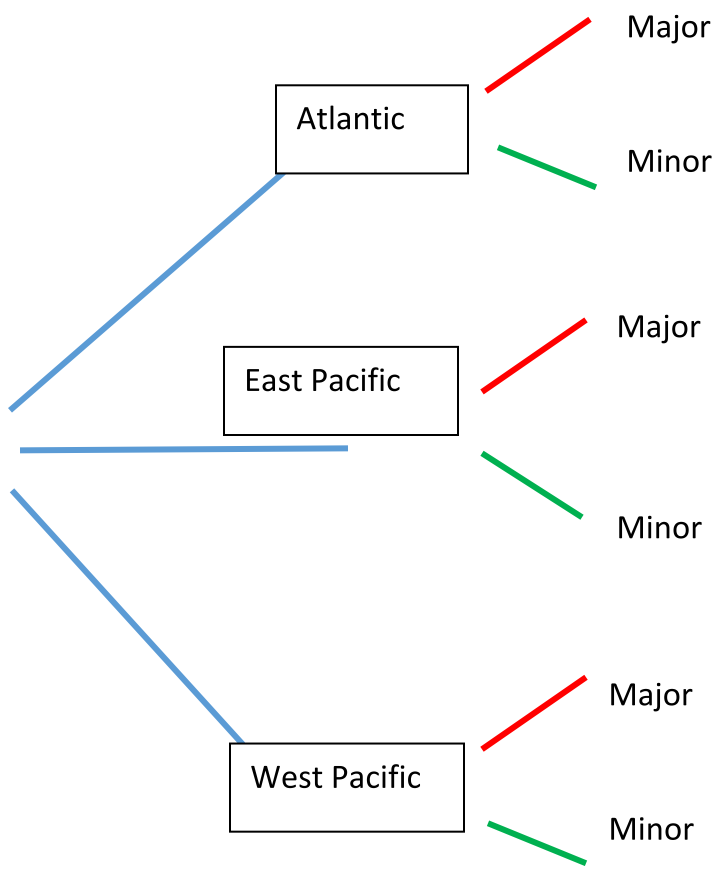
$$= 1 - (0.75)^4$$

10. A tropical storm is classified as major if it has sustained winds greater than 110 miles per hour. Based on data from the past two decades, a meteorologist estimated the following percentages about future storms.

- 20% of all tropical storms will originate in the Atlantic Ocean, of which 20% will be classified as major.
- 30% of all tropical storms will originate in the eastern Pacific Ocean, of which 15% will be classified as major.
- 50% of all tropical storms will originate in the western Pacific Ocean, of which 25% will be classified as major.

Based on the meteorologist's estimates, approximately what is the probability that a future tropical storm will originate in the Pacific Ocean and be classified as major?

A. 0.045
B. 0.125
C. 0.170
D. 0.400
E. 0.960



A. 0.045
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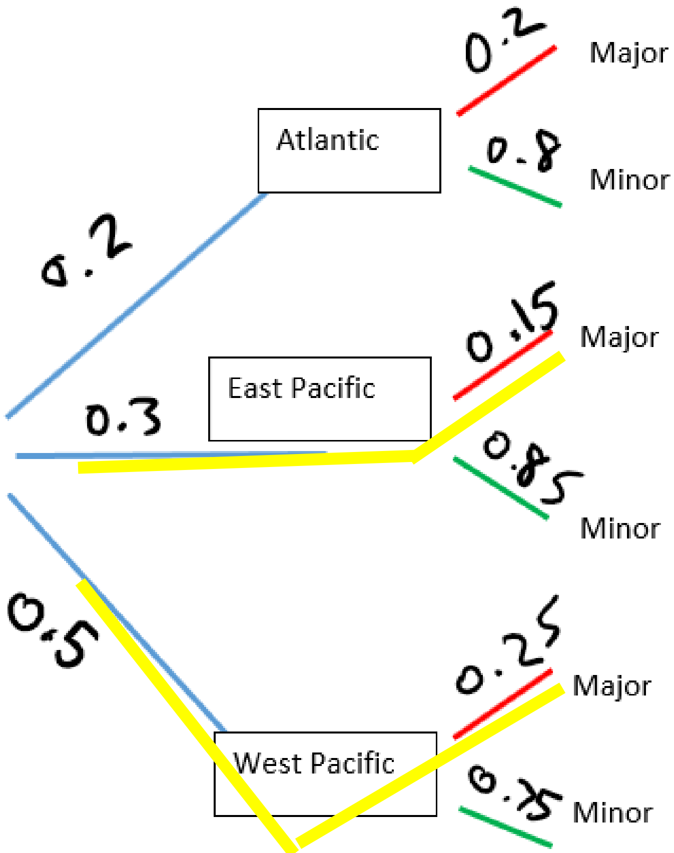


$$P(\text{originates in Pacific} \cap Major) = P(E\ Pacific \cap Major) + P(W\ Pacific\ \cap Major)$$

$$= (0.3) \cdot (0.15) + (0.5)(0.25) = \mathbf{0.170}$$

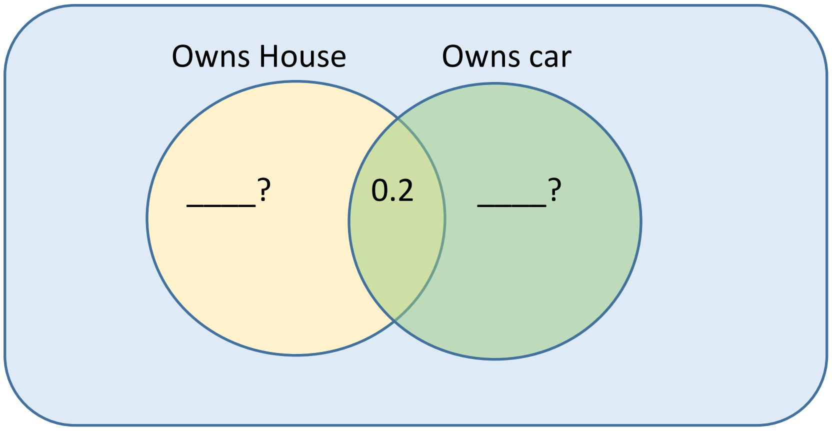
Other possible questions

P(originates in West pacific | Major storm) =

P(Minor storm | originates in East pacific) =

11. A marketing survey indicates that 60% of the population owns an automobile, 30% owns a house, and 20% owns both an automobile and a house. Calculate the probability that a person chosen at random owns an automobile or a house, but not both.

a. 0.4 b. 0.5 c. 0.6 d. 0.7 e. 0.9



$P(\text{House}) = 0.30$ $P(\text{car}) = 0.60$

$P(House\ \cap not\ car) =$ $P(car\ \cap not\ House) =$

$P(House\ \cap not\ car) = 0.1$ $P(car\ \cap not\ House) = 0.4$

12.. The probability that a visit to a primary care physician's (PCP) office results in neither lab work nor referral to a specialist is 35%. Of those coming to a PCP's office, 30% are referred to a specialist and 40% require lab work. Determine the probability that a visit to a PCP's office results in both lab work and referral to a specialist.

a. 0.05 b. 0.12 c. 0.18 d. 0.25 e. 0.35

$$P(no\ lab\ \cap no\ specialist) = 1 - P(lab\ \cup specialist)$$

$$P(none) = 1 - P(at\ least\ one)$$

$$0.35 = 1 - P(lab\ \cup specialist)$$

Use the following scenario to answer questions 13 to 15: A psychologist studied the number of puzzles that subjects were able to solve in a five-minute period while listening to soothing music. Let *X* be the number of puzzles completed successfully by a random chosen subject. The following probability distribution for *X* was found:

X	1	2	3	4
P(X)	0.2	0.4	0.3	0.1

13. What type of random variable is being defined? Is this a valid probability distribution? Why or why not?

This scenario describes the number of puzzles solved, which is a discrete random variable. YES, this is a valid probability distribution because all possible outcomes for the random variable *X* have probabilities from $0 < x_i < 1, and \ \sum P(x_i) = 1$

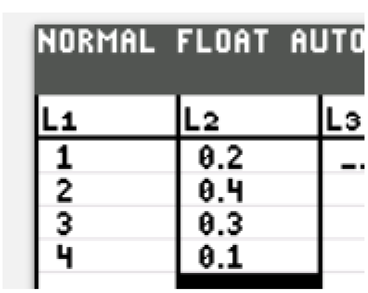
14. What is the probability that a randomly chosen subject completes more than the expected number of puzzles in the five-minute period?

You need to compute the expected value of the random variable:

$$\mu_x = E(X) = \sum x_i \cdot P(x_i)$$

<u>On the TI-84</u>: Go to STAT EDIT to enter data,

then go to STAT CALC to calculate the 1-variable statistics



The “sample mean” is really the expected value.

$$\bar{x} \ \ actually \ \mu_x = E(X) = 2.3$$

So the probability that a randomly chosen subject completes more than the expected value is: $P(X > x_i) = 2.3, which\ is\ really\ \ P(X > 2)\ or\ P(X \geq 3)$

$$\text{So } P(X \geq 3) = P(X = 3) + P(X = 4) = 0.3 + 0.1 = \mathbf{0.4}$$

Based upon the probability distribution for the random variable, there is a 40% probability that that a randomly chosen subject will complete more puzzles than the expected value of puzzles solved.

15. What are the values for σ_X and $\sigma_X^2\ \ or\ \ (\sigma_X)^2$ for the random variable X?

Probability Distribution	Mean	Standard Deviation
Discrete random variable, X	$\mu_X = E(X) = \Sigma x_i P(x_i)$	$\sigma_X = \sqrt{\Sigma(x_i - \mu_X)^2 P(x_i)}$

What does this formula mean?

So we need to find the sum of the products of each possible value of the variable's squared deviation from the expected value times its respective probability

FREE Response Question

A grocery store purchases melons from two distributors, J and K. Distributor J provides melons from organic farms. The distribution of the diameters of the melons from Distributor J is approximately normal with mean 133 millimeters (mm) and standard deviation 5 mm.

(a) For a melon selected at random from Distributor J, what is the probability that the melon will have a diameter greater than 137 mm?

Distributor K provides melons from nonorganic farms. The probability is 0.8413 that a melon selected at random from Distributor K will have a diameter greater than 137 mm. For all the melons at the grocery store, 70 percent of the melons are provided by Distributor J and 30 percent are provided by Distributor K.

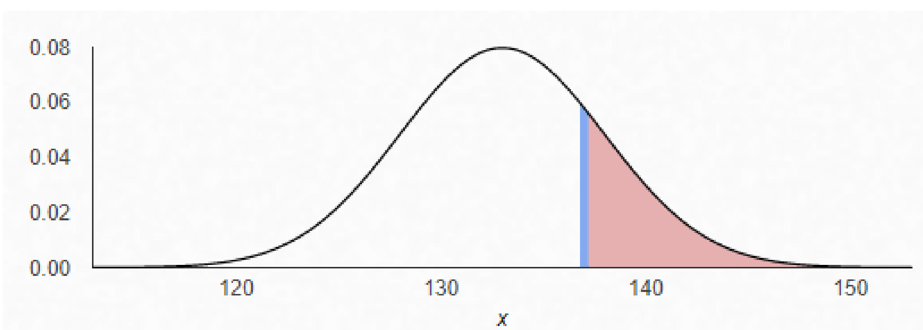
(b) For a melon selected at random from the grocery store, what is the probability that the melon will have a diameter greater than 137 mm?

(c) Given that a melon selected at random from the grocery store has a diameter greater than 137 mm, what is the probability that the melon will be from Distributor J?

Part A:

Event J = a melon selected from distributor J

$$P(J \cap diameter > 137) = ?$$

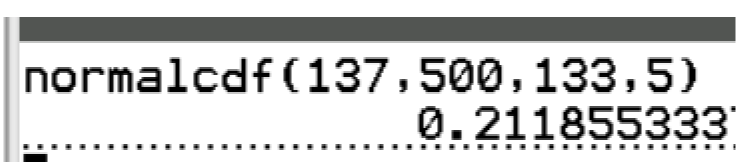


$$z\ score = \frac{137-133}{5} = 0.8$$

We want the probability that a melon chosen will be greater (a larger z score) than one with a diameter of 137 mm.

P(J melon is > 137) $= P(z > 0.8) = 1 - 0.7881 = \mathbf{0.2119}$

Also on TI-84, you could have entered:


normalcdf(137,500,133,5)
0.211855333

Part B: $P(any\ store\ melon\ has\ diameter > 137)$

$$P(K\ \cap diameter > 137) \cup P(J\ \cap diameter > 137) = ?$$

Which is

$$P(K\ \cap diameter > 137) = 0.30(0.8413) = 0.2524$$

$$P(J\ \cap diameter > 137) = 0.70(0.2119) = 0.1483$$

$$P(any\ store\ melon\ has\ diameter > 137) = 0.1483 + \ 0.2524 = \mathbf{0.4007}$$

There is just over a 40% chance that a randomly selected melon in the store will have a diameter greater than 137 mm.

Part C: $P(J\ |\ diameter > 137) = ?$

$$P(J\ |\ diameter > 137) = \frac{P(J \cap diam. > 137)}{P(diam. > 137)} = \frac{0.1483}{0.4007} = \mathbf{0.3701}$$

There is just over a 37% chance that a randomly selected melon in the store will have come from distributor J, given that it has a diameter > 137 mm

MC Problem Sets

Multiple Choice Questions – ANSWERS

Page 2

Probability 1: C

Probability 2: A

Probability 12: B

Probability 4: D

Page 3

Probability 23: D

Probability 15: D

Probability 21: C

Probability 16: A

Page 4

Probability 19: D

Probability 22: D

Probability 17: E

Probability 9: D

Note: Please explain solutions (on next pages)!

PROBABILITY 4

The following is from a particular region's mortality table.

Age	0	20	40	60	80
Number Surviving	10,000	9,700	9,240	7,800	4,300

What is the probability that a 20-year-old will survive to be 60?

(A) .1959 (B) .4419 (C) .7800 (D) .8041 (E) .9700

Conditional probability:

$$P(will\ survive\ to\ 60|survived\ to\ 20) = \frac{7800}{9700} \approx 0.8041$$

PROBABILITY 9

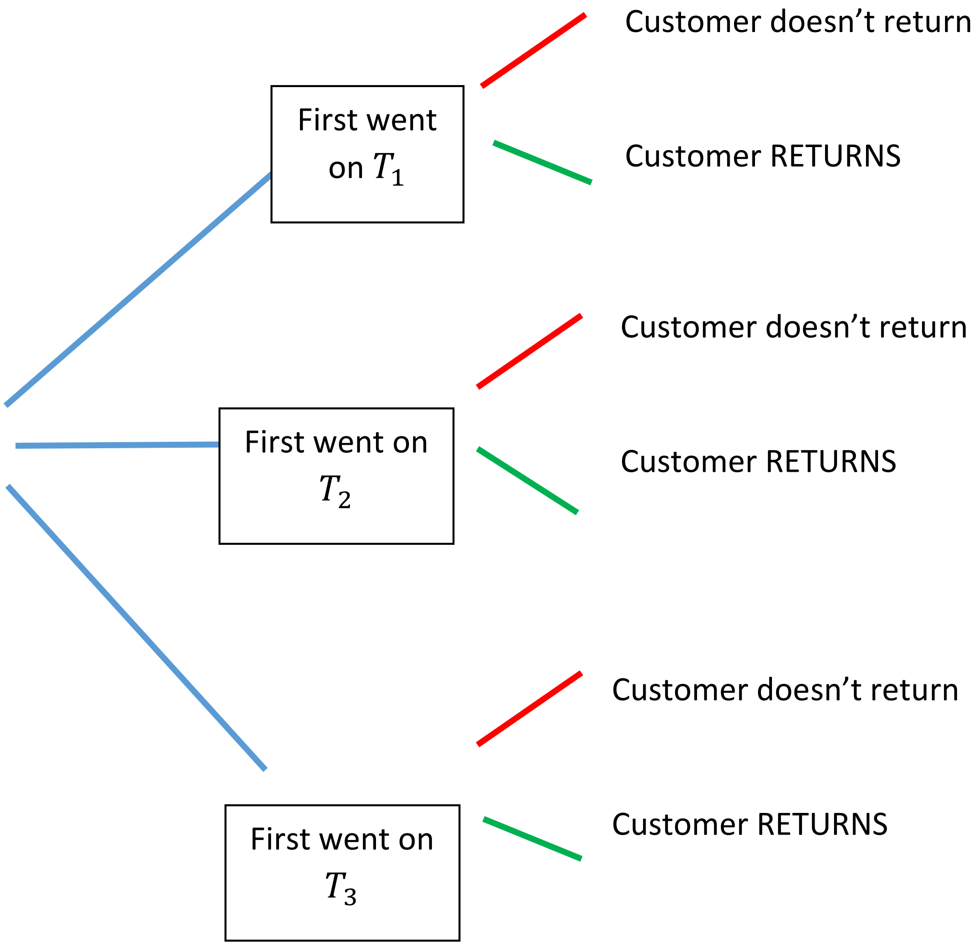
A travel agent books passages on three different tours, with half her customers choosing tour T_1, one-third choosing T_2, and the rest choosing T_3. The agent has noted that three-quarters of those who take tour T_1 return to book passage again, two-thirds of those who take T_2 return, and one-half of those who take T_3 return. If a customer does return, what is the probability that the person first went on tour T_2?

(A) 1/3 (B) 2/3 (C) 2/9 (D) 16/49 (E) 49/72

This is a conditional probability problem that is asking for this subset of the full sample:

$$P(\text{person first went on } T_2 \mid customer\ returns) = \frac{P(went\ on\ T_2 \cap returns)}{P(customer\ returns)}$$

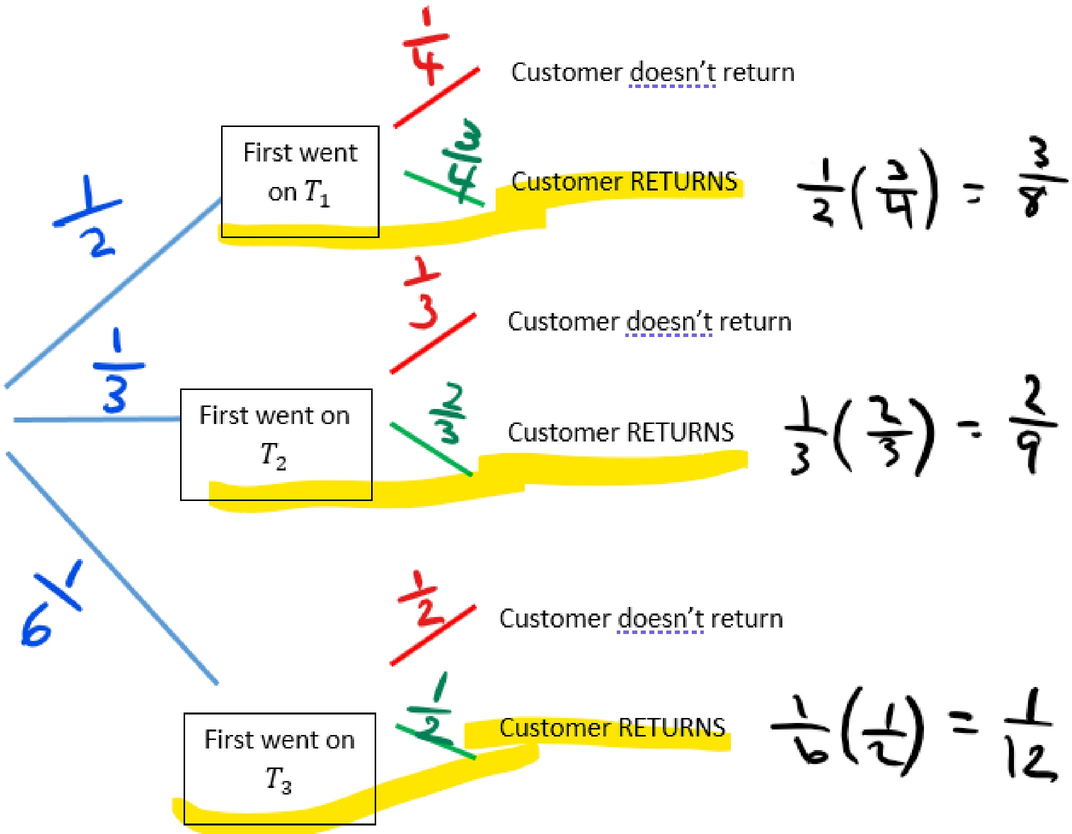
You can use a tree diagram with actual probabilities, or by considering a sample of x customers:



Answer to Probability #9

$P(\text{person first went on } T_2 \mid customer\ returns) = \frac{P(went\ on\ T_2)}{P(customer\ returns)}$

You can use a tree diagram with actual probabilities, or by considering a sample of x customers:



So $P(\text{person first went on } T_2 \mid customer\ returns) = \frac{P(went\ on\ T_2)}{P(customer\ returns)}$

$$\frac{P(went\ on\ T_2 \cap returns)}{P(customer\ returns)} = \frac{\frac{2}{9}}{\frac{3}{8}+\frac{2}{9}+\frac{1}{12}} = \frac{16}{49}$$

PROBABILITY 23

According to one poll, only 8 percent of the public say they "trust Congress." In a simple random sample of ten people, what is the probability that at least one person "trusts Congress"?

(A) .188 (B) .378 (C) .434 (D) .566 (E) .622

Event A = someone who "trusts Congress"

$A^C = someone\ who\ does\ NOT\ "trust\ Congress"$

According to poll, $P(A) = 0.08$

So knowing the complement rule, $P(A^C) = 0.92$

Assuming independence (since we are using SRS), then probability of

$$P(at\ least\ one\ A) = 1 - P(none\ A)$$

$$= 1 - (0.92)^{10}$$

$$= 0.56561154 \approx \mathbf{0.566}$$