Chapter 6: Probability: What are the Chances?

Section 6.1- Chance Experiments & Events

Probability Rules

Adapted from The Practice of Statistics, STARNES YATES, MOORE, 4th edition – For AP*

Chapter 6
Probability: What Are the Chances?

- 6.1 Chance Experiments & Events
- **6.2 Probability Rules**
- 6.3 Conditional Probability and Independence

Warm-UP

- Take a copy of the half sheet, A Physician's Thinking and carefully read the scenario that begins the "To facilitate early detection of breast cancer…"
- Write down in your notes/warm-ups the **specified population** in the scenario
- Write down specifically what you are trying to find.
- Wait to obtain the $P(A \mid B)$ from Mr. L…Work it out and be ready to share your process for finding a solution
- Later: Take a copy of the half sheet with example "Who has pierced ears?" Be prepared to complete the missing information.

Warm-UP

- The specified population in the scenario:

  Asymptomatic women aged 40 to 50 who participate in mammography screening

- What you are trying to find?

  What is the probability that **she** actually has breast cancer?

*Given a woman from this pop. that **tests positive**, what is the probability that **she** actually has breast cancer?*

What is the probability that a woman who tests positive for breast cancer actually has breast cancer?

A. 99%
B. 93%
C. 90%
D. 50%
E. 9%

Warm-UP: *One possible* process for the solution

- Consider any 1000 women from the population
- Given 0.8% incident rate for breast cancer
- of these 1000: 8 w/ cancer 992 without
- test results:

8 w/ cancer		992 without	
0.9 Pos	0.1 FALSE neg.	0.93 Neg	0.07 FALSE pos.
≈ 7	≈ 1	≈ 922	≈ 70

Therefore, only 7 out 77 women who had a positive result should expect to actually have breast cancer ($\approx 9\%$)

Section 6.2
Probability Rules

Learning Objectives

After this section, you should be able to…

- ✓ DESCRIBE chance behavior with a probability model
- ✓ DEFINE and APPLY basic rules of probability
- ✓ DETERMINE probabilities from two-way tables
- ✓ CONSTRUCT Venn diagrams and DETERMINE probabilities

Probability Models

In Section 5.1, we used simulation to imitate chance behavior. Fortunately, we don't have to always rely on simulations to determine the probability of a particular outcome.

Descriptions of chance behavior contain two parts:

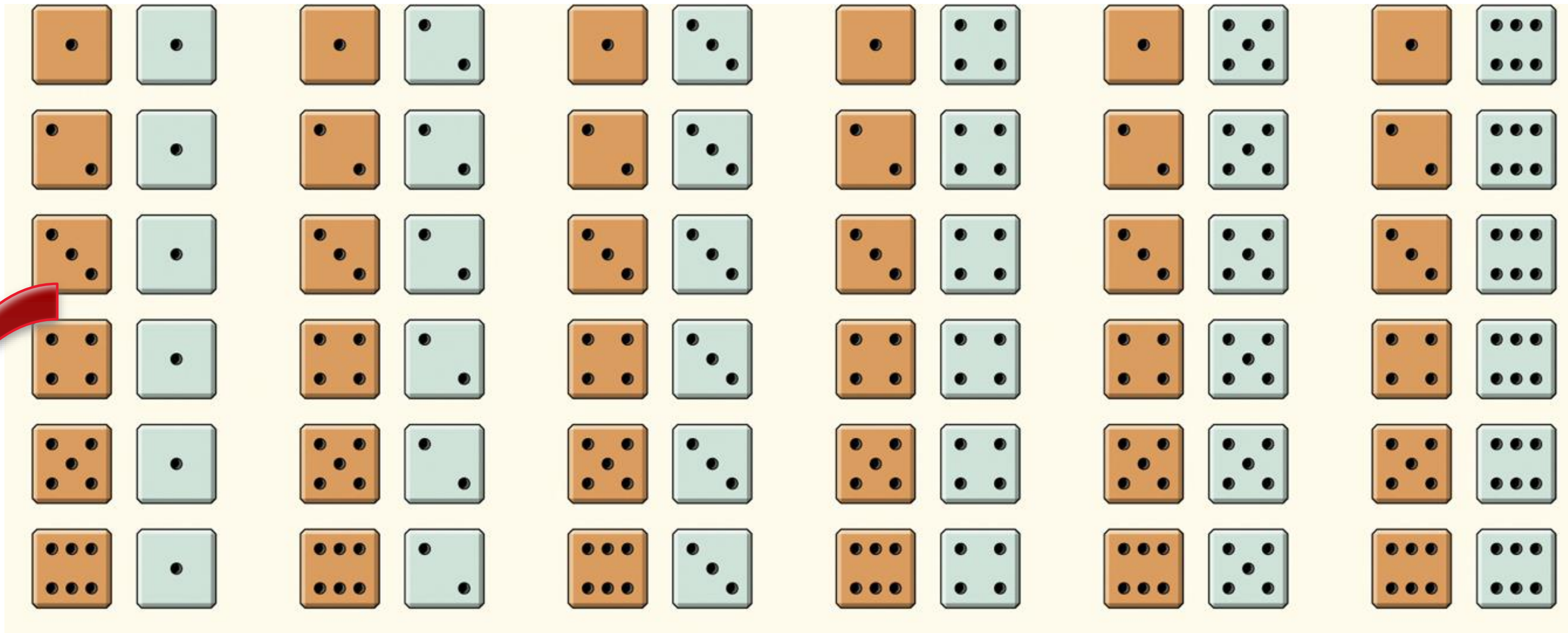
Definition:

The **sample space *S*** of a chance process is the set of all possible outcomes.

A **probability model** is a description of some chance process that consists of two parts: a sample space *S* and a probability for each outcome.

Example: Roll the Dice

Give a probability model for the chance process of rolling two fair, six-sided dice – one that's red and one that's green.



Sample Space 36 Outcomes

Since the dice are fair, each outcome is equally likely. Each outcome has probability 1/36.

Probability Models

Probability models allow us to find the probability of any collection of outcomes.

Definition:

An **event** is any collection of outcomes from some chance process. That is, an event is a subset of the sample space. Events are usually designated by capital letters, like *A, B, C,* and so on.

If *A* is any event, we write its probability as P(*A*).

In the dice-rolling example, suppose we define event *A* as "sum is 5."

There are 4 outcomes that result in a sum of 5.
Since each outcome has probability 1/36, P(*A*) = 4/36.

Suppose event *B* is defined as "sum is not 5." What is P(*B*)? *P*(*B*) = 1 – 4/36
= 32/36

Basic Rules of Probability

All probability models must obey the following rules:

- **The probability of any event is a number between 0 and 1.**
- **All possible outcomes together must have probabilities whose sum is 1.**
- **If all outcomes in the sample space are equally likely, the probability that event *A* occurs can be found using the formula**

$$P(A) = \frac{\text{number of outcomes corresponding to event } A}{\text{total number of outcomes in sample space}}$$

- **The probability that an event does not occur is 1 minus the probability that the event does occur.**
- **If two events have no outcomes in common, the probability that one or the other occurs is the sum of their individual probabilities.**

Definition:

Two events are **mutually exclusive (disjoint)** if they have no outcomes in common and so can never occur together.

Basic Rules of Probability

- For any event A, $0 \le P(A) \le 1$.
- If S is the sample space in a probability model,

$$P(S) = 1.$$

- In the case of equally likely outcomes,

$$P(A) = \frac{\text{number of outcomes corresponding to event } A}{\text{total number of outcomes in sample space}}$$

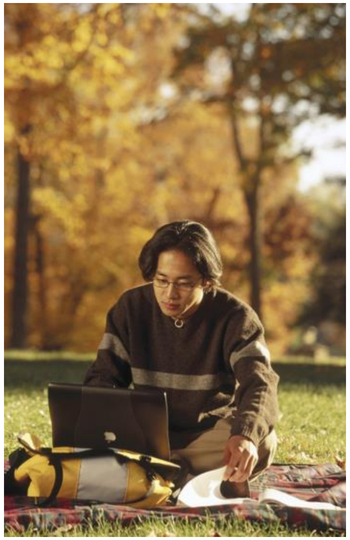
- **Complement rule:** $P(A^C) = 1 - P(A)$
- **Addition rule for mutually exclusive events:** If A and B are mutually exclusive,

$$P(A \text{ or } B) = P(A) + P(B).$$

Example: Distance Learning

Distance-learning courses are rapidly gaining popularity among college students. Randomly select an undergraduate student who is taking distance-learning courses for credit and record the student's age. Here is the probability model:

Age group (yr):	18 to 23	24 to 29	30 to 39	40 or over
Probability:	0.57	0.17	0.14	0.12



(a) Show that this is a legitimate probability model.

Each probability is between 0 and 1 and
0.57 + 0.17 + 0.14 + 0.12 = 1

(b) Find the probability that the chosen student is not in the traditional college age group (18 to 23 years).

$$\boldsymbol{P(\text{not 18 to 23 years}) = 1 - P(\text{18 to 23 years}) = 1 - 0.57 = 0.43}$$

Two-Way Tables and Probability

When finding probabilities involving two events, a two-way table can display the sample space in a way that makes probability calculations easier.

Consider the example for students with pierced ears. Suppose we choose a student at random. Find the probability that a chosen student

	Pierced Ears?		
Gender	**Yes**	**No**	**Total**
Male	19	71	**90**
Female	84	4	**88**
Total	**103**	**75**	**178**

(a) has pierced ears.

(b) is a male with pierced ears.

(c) is a male or has pierced ears.

Define events *A*: is male and *B*: has pierced ears.

(a) Each student is equally likely to be chosen. 103 students have pierced ears. So, *P*(pierced ears) = *P*(*B*) = 103/178.

(b) We want to find *P*(male and pierced ears), that is, *P*(*A* and *B*). Look at the intersection of the "Male" row and "Yes" column. There are 19 males with pierced ears. So, *P*(*A* and *B*) = 19/178.

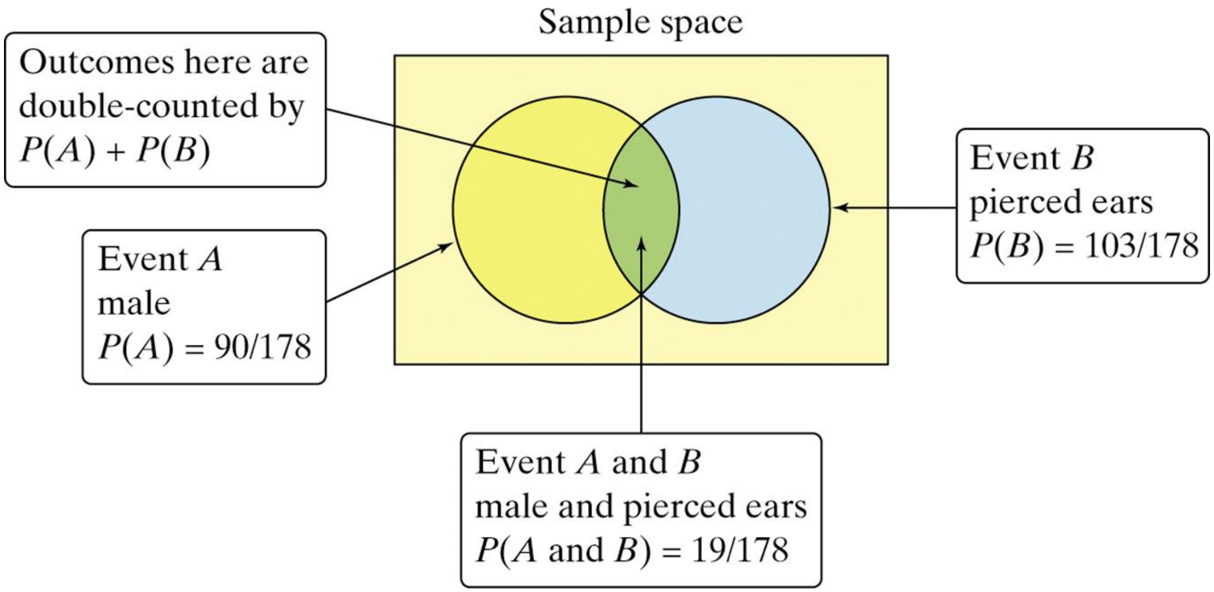
(c) We want to find *P*(male *or* pierced ears), that is, *P*(*A* or *B*). There are 90 males in the class and 103 individuals with pierced ears. However, 19 males have pierced ears – don't count them twice! *P*(*A* or *B*) = (19 + 71 + 84)/178. So, *P*(*A* or B) = 174/178

Probability Rules

Two-Way Tables and Probability

Note, the previous example illustrates the fact that we can't use the basic addition rule for mutually exclusive events unless the events have no outcomes in common.

The **Venn diagram** below illustrates why.



General Addition Rule for Two Events

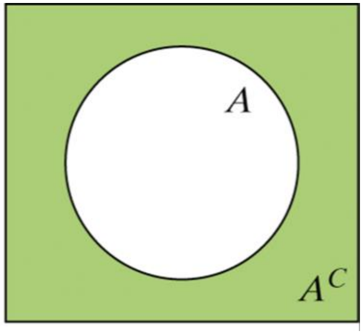
If A and B are any two events resulting from some chance process, then

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

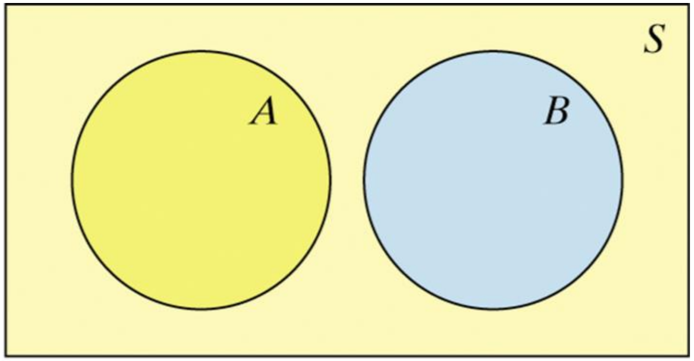
Venn Diagrams and Probability

Because Venn diagrams have uses in other branches of mathematics, some standard vocabulary and notation have been developed.

The complement A^C contains exactly the outcomes that are not in *A*.

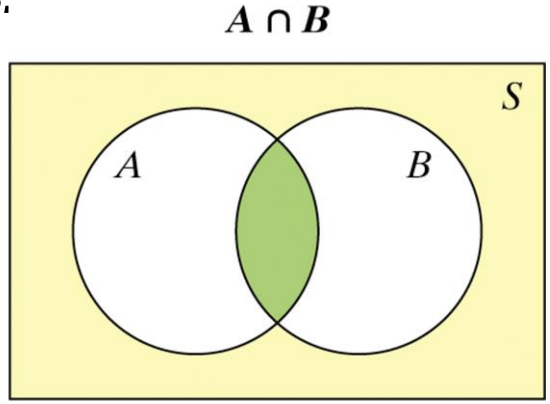


The events *A* and *B* are mutually exclusive (disjoint) because they do not overlap. That is, they have no outcomes in common.

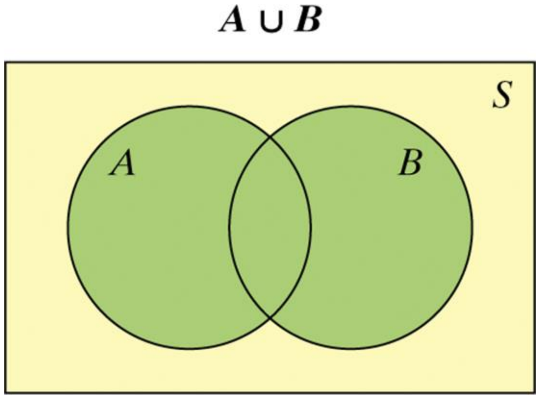


Venn Diagrams and Probability

The intersection of events *A* and *B* (*A* ∩ *B*) is the set of all outcomes in both events *A and B*.



The union of events *A* and *B* (*A* ∪ *B*) is the set of all outcomes in either event *A or B*.

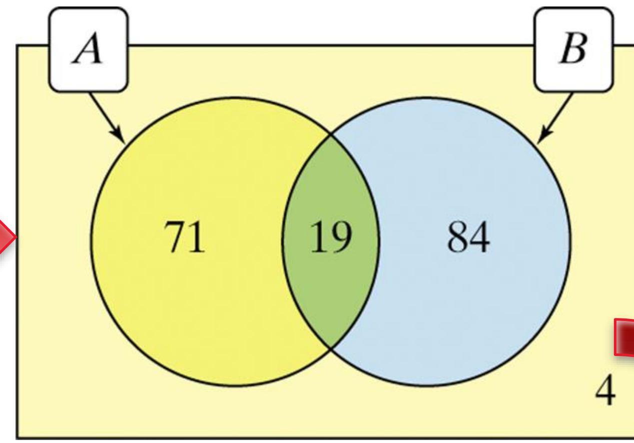


Hint: To keep the symbols straight, remember ∪ for union and ∩ for intersection.

Venn Diagrams and Probability

Recall the example on gender and pierced ears. We can use a Venn diagram to display the information and determine probabilities.

	Pierced Ears?		
Gender	**Yes**	**No**	**Total**
Male	19	71	**90**
Female	84	4	**88**
Total	**103**	**75**	**178**



Define events *A*: is male and *B*: has pierced ears.

Region in Venn diagram	In words	In symbols	Count
In the intersection of two circles	Male and pierced ears	$A \cap B$	19
Inside circle A, outside circle B	Male and no pierced ears	$A \cap B^C$	71
Inside circle B, outside circle A	Female and pierced ears	$A^C \cap B$	84
Outside both circles	Female and no pierced ears	$A^C \cap B^C$	4

Section 6.2
Probability Rules

Summary

In this section, we learned that…

- ✓ A **probability model** describes chance behavior by listing the possible outcomes in the **sample space *S*** and giving the probability that each outcome occurs.
- ✓ An **event** is a subset of the possible outcomes in a chance process.
- ✓ For any event A, $0 \le P(A) \le 1$
- ✓ $P(S) = 1$, where S = the sample space
- ✓ If all outcomes in S are equally likely, $P(A) = \dfrac{\text{number of outcomes corresponding to event } A}{\text{total number of outcomes in sample space}}$
- ✓ $P(A^C) = 1 - P(A)$, where A^C is the **complement** of event A; that is, the event that A does not happen.

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Section 6.2 & 6.3
Probability Rules

Summary

In this section, we learned that…

- ✓ Events A and B are **mutually exclusive (disjoint)** if they have no outcomes in common. If A and B are disjoint, $P(A \text{ or } B) = P(A) + P(B)$.
- ✓ A **two-way table** or a **Venn diagram** can be used to display the sample space for a chance process.
- ✓ The **intersection ($A \cap B$)** of events A and B consists of outcomes in both A and B.
- ✓ The **union ($A \cup B$)** of events A and B consists of all outcomes in event A, event B, or both.
- ✓ The **general addition rule** can be used to find $P(A \text{ or } B)$:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Looking Ahead…

<u>In the next Section…</u>

We’ll learn how to calculate conditional probabilities as well as probabilities of independent events.

We’ll learn about

- ✓ **Conditional Probability**
- ✓ **Independence**
- ✓ **Tree diagrams and the general multiplication rule**
- ✓ **Special multiplication rule for independent events**
- ✓ **Calculating conditional probabilities**