Prob & Stats HW #8: Probability Test REVIEW

1. How is the conditional probability that the event E occurs given that event F has already occurred different from the conditional probability that the event F occurs given that event E has already occurred?

   Prob. of E, given **F** is $P(E|F) = \frac{P(E\cap F)}{P(F)}$,

   but the Prob. of **F**, given **E** is $P(F|\,E) = \frac{P(E\cap F)}{P(E)}$

2. If P(A) = 0.5, P(B) = 0.3, and P(A∩B) = 0.15, then:
   a. A and B are disjoint events
   b. A and B are dependent events
   c. A and B are independent events
   d. A and B are not disjoint
   e. A and B are neither disjoint nor independent

3. If a fair coin is flipped three times with the outcome of each flip independent of each other, then the probability that at least one of the three flips results in a head is what?

   ANS: $P(at\ least\ 1\ head) = P(no\ heads)^C = 1 - (0.5)^3 = \mathbf{0.875}$

4. Only 20% of the applicants for new positions at a large software company are female. Assuming that two positions will be filled independently of each other, what is the probability that both positions are filled by females?

   ANS: $0.2 \cdot (0.2) = \mathbf{0.04}$

5. A family is going shopping for a new van. The probability that the family will purchase a Ford van is 0.33, a Chevy van is 0.25, a Dodge van 0.20, and a Toyota van 0.22. The probability that the family purchases a Toyota van or a Ford van or a Chevy van is:
   a. $.33 \times .22 \times .25$
   b. $1 - .33 \times .22 \times .25$
   c. $.33 + .22 + .25$
   d. $1 - (.33 + .22 + .25)$
   e. None of the above

6. Suppose there are 60 students in a statistics class of which 24 are female. If three students are selected without replacement to work problems at the board, what is the probability that all three of the students chosen are female?

   ANS: $\frac{24}{60} \times \frac{23}{59} \times \frac{22}{58} = \frac{506}{8555} \approx \mathbf{0.0591}$

7. The probability that a new tire will have a blowout in the first year is 0.10. If the four tires on a new car function independently of each other, what is the probability that at least one tire blows out in the first year?

a. $.10 \times .10 \times .10 \times .10$

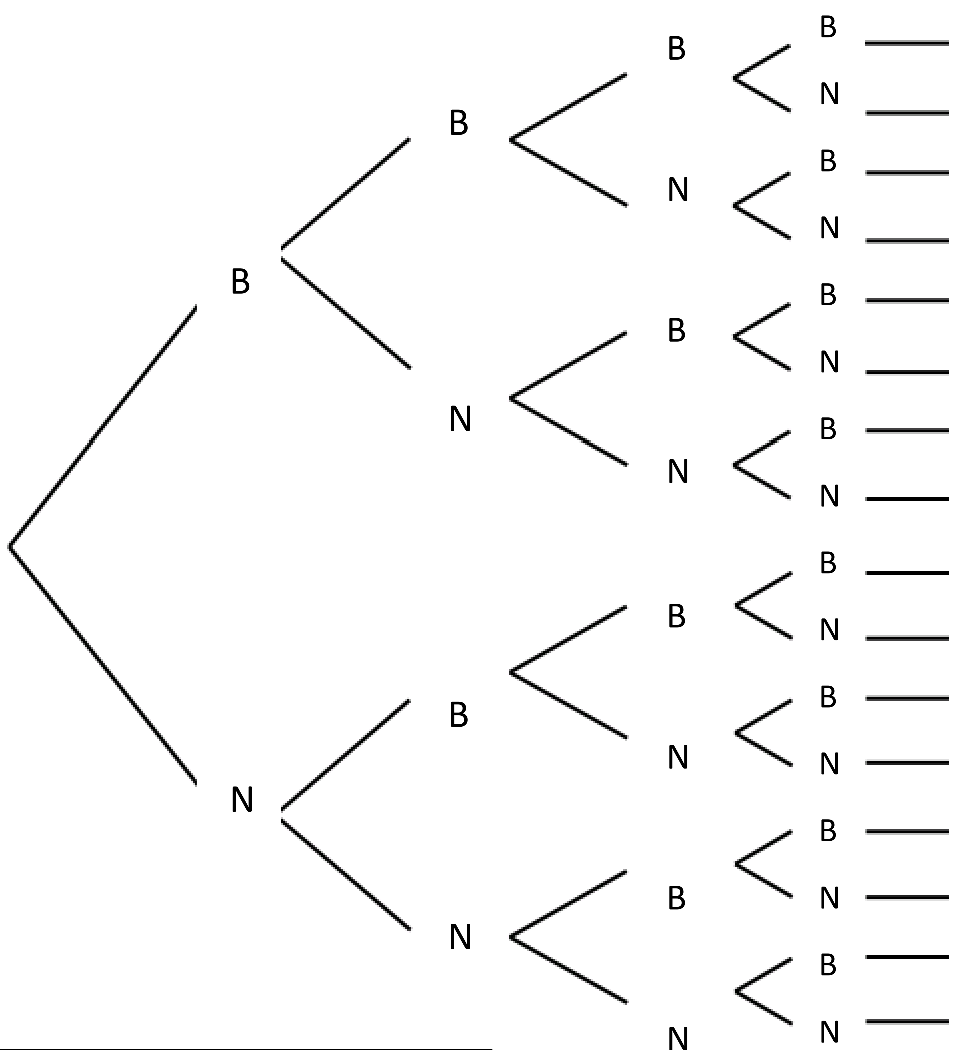
b. $1 - (.10 \times .10 \times .10 \times .10)$

c. $.90 \times .90 \times .90 \times .90$

d. $1 - (.90 \times .90 \times .90 \times .90)$

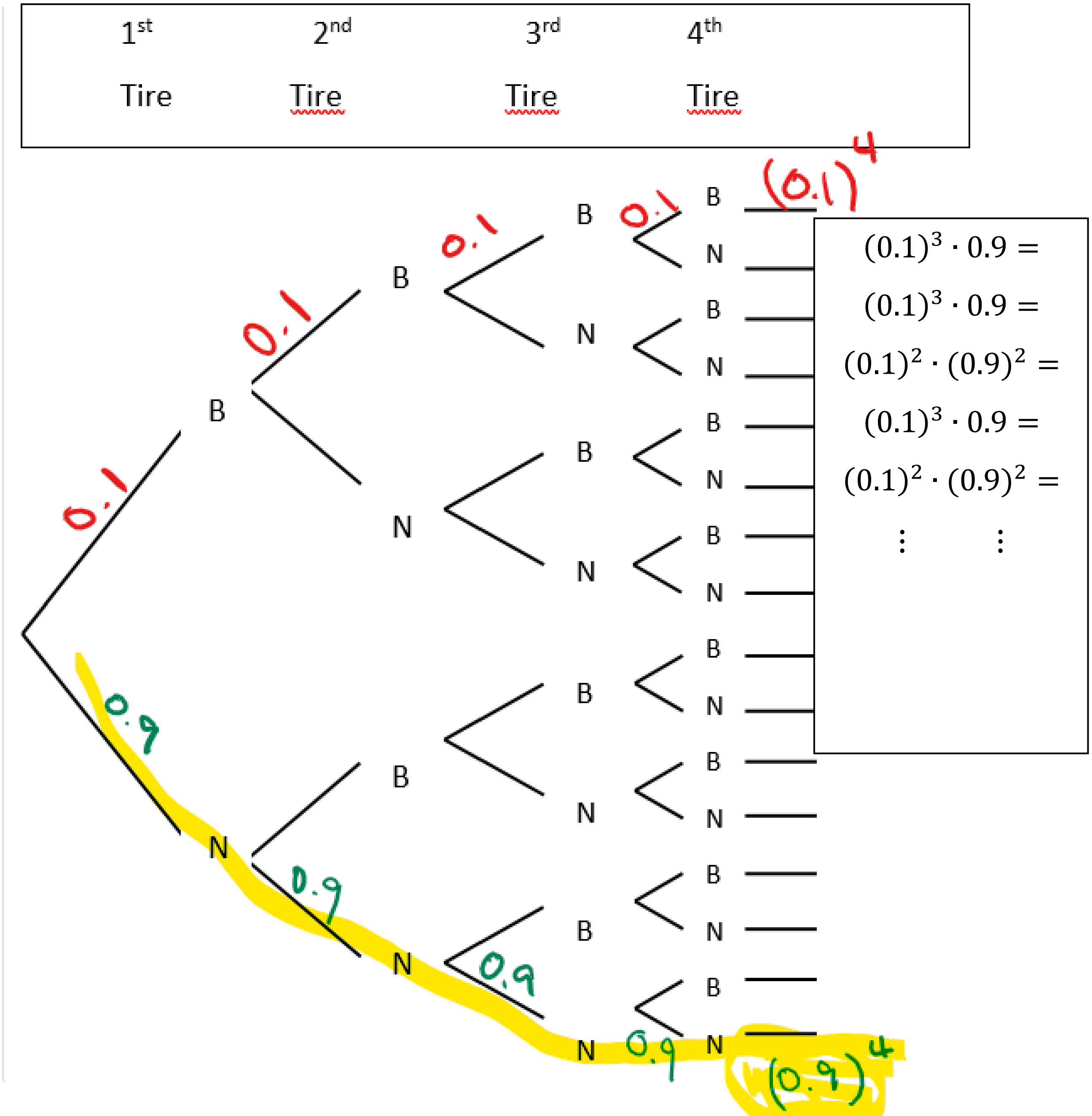
e. None of the above

1st Tire	2nd Tire	3rd Tire	4th Tire



Tire has a Blowout = B

No blowout = N



$(0.1)^3 \cdot 0.9 =$

$(0.1)^3 \cdot 0.9 =$

$(0.1)^2 \cdot (0.9)^2 =$

$(0.1)^3 \cdot 0.9 =$

$(0.1)^2 \cdot (0.9)^2 =$

$\vdots \quad \vdots$

Tire has a Blowout = B 0.1

No blowout = N = 0.9

P(at least one tire has blowout) = 1 – P(NO tires have blowout)

8. A complex electronic device contains three components, A, B, and C. The probabilities of failure for each component in any one year are 0.01, 0.03, and 0.04, respectively. If any one component fails, the device will fail. Assuming the components fail independently of one another, what is the probability that the device will **not** fail in one year?

A. Less than 0.01
B. 0.078
C. 0.080
D. 0.922
E. Greater than 0.99

P(Success part A) = 0.99 $P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$

P(Success part B) = 0.97 $P(A \cap B \cap C) = 0.99 \cdot 0.97 \cdot 0.96$

P(Success part C) = 0.96

9. A large store has a customer service department where customers can go to ask for help with store-related issues. According to store records, approximately twenty-five percent of all customers who go to the service department ask for help finding an item. Assume the reason each customer goes to the service department is independent from customer to customer. Based on the approximation, what is the probability that at least 1 of the next 4 customers who go to the service department will ask for help finding an item?

A. $4\left(\frac{1}{4}\right)^2$

B. $1-\left(\frac{1}{4}\right)^2$

C. $1-\left(\frac{1}{4}\right)^4$

D. $1-\left(\frac{3}{4}\right)^4$

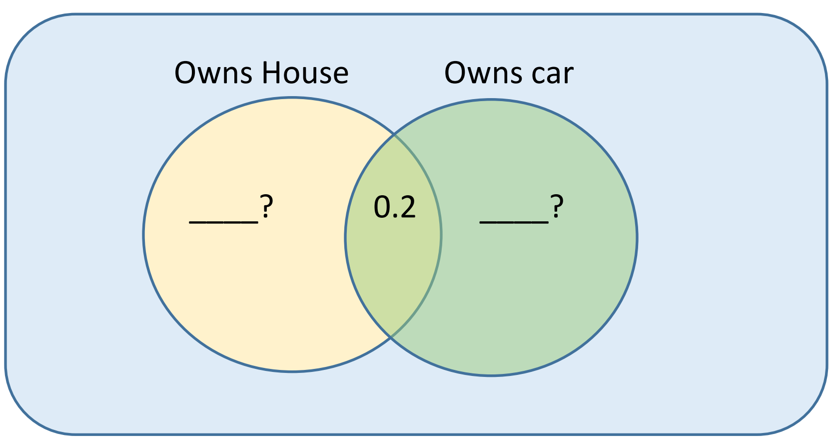
E. $\left(\frac{4}{4}\right)\left(\frac{3}{4}\right)\left(\frac{2}{4}\right)\left(\frac{1}{4}\right)$

ANS: P(at least one asks HELP) = 1 – P(NO one asks HELP)

$$= 1 - (0.75)^4$$

10. A marketing survey indicates that 60% of the population owns an automobile, 30% owns a house, and 20% owns both an automobile and a house. Calculate the probability that a person chosen at random owns an automobile or a house, but not both.

a. 0.4 b. 0.5 c. 0.6 d. 0.7 e. 0.9



$P(\text{House}) = 0.30$ $P(\text{car}) = 0.60$

$P(House\ \cap not\ car) =$ $P(car\ \cap not\ House) =$

$P(House\ \cap not\ car) = 0.1$ $P(car\ \cap not\ House) = 0.4$

12. The probability that a visit to a primary care physician's (PCP) office results in neither lab work nor referral to a specialist is 35%. Of those coming to a PCP's office, 30% are referred to a specialist and 40% require lab work. Determine the probability that a visit to a PCP's office results in both lab work and referral to a specialist.

    a. 0.05 b. 0.12 c. 0.18 d. 0.25 e. 0.35

13.When is a data value is called an outlier? How is it calculated?

If it is unusually large or small compared to the rest of the data.

Calculated outlier: $Outlier = 1.5(IQR)\ above\ Q_3$ or $1.5(IQR)\ below\ Q_1$

14. A study of voting chose 663 registered voters at random shortly after an election. Of these, 56% said they had voted in the election. Election records show that only 61% of registered voters voted in the election. Which of the following states is true about this situation?

A. 61% is a sample, 56% is a population
B. 61% and 56% are both statistics
C. *61% is a statistic and 56% is a parameter*
D. 61% is a parameter and 56% is a statistic
E. 61% and 56% are both parameters

15. What are the measures of center and spread (or variability) in a data set? How are they paired?

Center and spread (pairs):

- Median and IQR
- ***Mean*** and ***standard deviation***

16. What is a z-score and how do you calculate it?

This is a **measure of position** for a value or observation within a data set. It tells you **distance** (in **standard deviations**) and direction (above (+) or below(-)) the mean

$$z\ score = \frac{observ - mean}{S.D.}$$

17. What are percentiles that correspond to the z-scores below:

a) $z = 0.38$

a. equals a percentile of 0.6480 or almost the *sixty-fifth* percentile

b) $z = -1.64$

b) equals a percentile of 0.0505 or just larger than the *fifth* percentile

c) $z = 1.18$

c) equals a percentile of 0.8810 or approx. the *eighty-eighth* percentile

$d)\ z = 0$

d) equals a percentile of 0.5000 or exactly the fiftieth percentile (50%)

18. For the given percentiles, what are the z-scores that correspond to these:

a) percentile as a decimal = 0.0281 $\quad$ $z\text{-}score = -1.91$

b) percentile as a decimal = 0.3936 $\quad$ $z\text{-}score = -0.27$

c) percentile as a decimal = 0.7123 $\quad$ $z\text{-}score = 0.56$

d) percentile as a decimal = 0.9920 $\quad$ $z\text{-}score = 2.41$

e) 85% $\quad$ *z-score* b/t 1.03 and 1.04, or $z\text{-}score = 1.035$

f) 53% $\quad$ *z-score* b/t 0.07 and 0.08, or $z\text{-}score = 0.075$

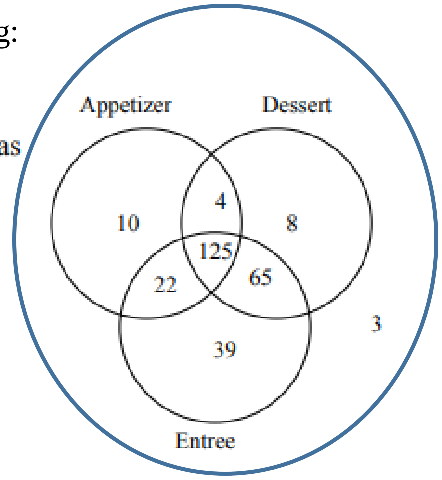
g) 9% $\quad$ *z-score* b/t -1.34 and -1.35, or $z\text{-}score = -1.341$

19. How do you find the union of events? How do you find the intersection of events?

You should KNOW!

20. Use the scenario and the Venn diagram to answer the following:

On a cruise ship's restaurant, a diner can have an appetizer, entrée, and dessert, although he can choose to only have one of them or two of them as shown by the Venn diagram to the right. Based on the Venn diagram, answer the following questions.



$P\ (A) = \frac{161}{276}$, $P\ (D) = \frac{202}{276}$ $P\ (E) = \frac{251}{276}$

a) P(a person orders all three)

b) P(a person only orders one of them)

c) P(a orders just an entrée and dessert)

d) P(a person orders two of them)

g) P (a person orders dessert | he orders an entree)

h) P (a person orders appetizer | he orders an entree)

g) P (a person orders an entree | he orders dessert)

h) P (a person orders dessert | he orders two of them)

j) Are ordering an appetizer and dessert independent? Explain?

$a)\ P(A \cap D\ \cap E) = \frac{125}{276}$

b) $P(only\ 1\ dish) = \frac{57}{276}$

c) $P(D\ \cap E) = \frac{65}{276}$

d) $P(\text{any two dishes}) = \frac{91}{276}$

e) $P(D \mid E) = \frac{190}{251}$

f) $P(A \mid E) = \frac{147}{251}$

21. Use the scenario to **create a Venn diagram** and then to answer the following questions:

In an apartment building having 3 vacant apartments, 45 apartments have a landline. 82 apartments have people with a cellphone, and 40 have people with both. Make a Venn Diagram and answer the probability questions.

Venn diagram

$$P(\mathrm{L} \cap C) = \frac{40}{90}$$

$$P(\mathrm{L} \mid C) = \frac{40}{82}$$

a) P(an apartment has a landline and a cellphone)

b) P(an apartment has a landline or a cellphone)

c) P(an apartment has a landline but not a cellphone)

d) P(an apartment has a cellphone but no landline)

e) P(an apartment has neither landline nor cellphone)

f) P(an apt. has a landline or cellphone but not both)

g) P (an apt has a cellphone | it has a landline)

h) P (an apt has a landline | it has a cellphone)

i) Are having a landline and cellphone disjoint? Explain.

22. What is the probability of selecting three cards from a deck of 52 that are:

a) All diamonds - Note: we could write $P(all\ 3\ cards\ are\ diamonds) = ?$

$$P(all\ 3\ cards\ are\ diamonds) = \frac{13}{52} \cdot \frac{12}{51} \cdot \frac{11}{50} = \mathbf{\frac{11}{850}} \approx \mathbf{0.0129}$$

b) exactly 2 cards are red $P(exactly\ 2\ Jacks) = \frac{4}{52} \cdot \frac{3}{51} \cdot \frac{48}{50} \cdot 3 = \frac{72}{5525} \approx \mathbf{0.013}$

c) all 3 cards are Jacks

d) a pair of any kind (i.e., a pair of aces, or a pair of eights)

23. Babies' health at birth: An **Apgar score** is a quick assessment of a baby's health within the first few minutes after being born. It measures five attributes: (1) Breathing effort. (2) Heart rate, (3) Muscle tone, (4) Reflexes, and (5) Skin color. Each category is scored with 0, 1, or 2, depending on the observed condition.

Define our random variable to be ***A*** for the Apgar score of a randomly selected baby one minute after birth. A nearly complete probability distribution is provided below.

Value of A	0	1	2	3	4	5	6	7	8	9	10
$P(A)$	0.001	0.006	0.007	0.008	0.012	0.020	0.038	0.099	0.319	0.435	?

a) What must the probability value be for an Apgar score of 10 for this to be a valid probability distribution?
b) Make a histogram of the distribution and then describe it.

c) Find the following probabilities for the random variable A:

i) $P(A = 6 \cup 7 \cup 8)$

ii) $P(A < 7)$

iii) $P\ (A \leq 7)$

iv) $P(5 \leq A < 9)$

Open Response Question

A grocery store purchases melons from two distributors, J and K. Distributor J provides melons from organic farms. The distribution of the diameters of the melons from Distributor J is approximately normal with mean 133 millimeters (mm) and standard deviation 5 mm.

(a) For a melon selected at random from Distributor J, what is the probability that the melon will have a diameter greater than 137 mm?

Distributor K provides melons from nonorganic farms. The probability is 0.8413 that a melon selected at random from Distributor K will have a diameter greater than 137 mm. For all the melons at the grocery store, 70 percent of the melons are provided by Distributor J and 30 percent are provided by Distributor K.

(b) For a melon selected at random from the grocery store, what is the probability that the melon will have a diameter greater than 137 mm?

(c) Given that a melon selected at random from the grocery store has a diameter greater than 137 mm, what is the probability that the melon will be from Distributor J?