Chapter 4: Numerical Methods for Distributions of Data

Interpreting Center & Variability in a Distribution

Adapted from Statistics and Data Analysis, 5th edition - For AP*

PECK, OLSEN, & DEVORE

Oct 17, 2022: Warm - UP

1) Given a fairly symmetric distribution that has a mean of 100 and a standard deviation of 15, what are the following z-scores:

- A) for an observation that is 110
- B) for an observation that is 85
- C) for an observation that is 142

2) Using the information from the problem above, what is the value of the observation that has a z-score: a) $z = -2$; b) $z = 1.58$

Oct. 2022: Warm-UP Answers

1) Given a fairly symmetric distribution that has a mean of 100 and a standard deviation of 15, what are the following z-scores:

$$z\ score = \frac{x\ - mean}{standard\ deviation}$$

- A) for an observation that is 110

$$z\ score = \frac{110\ - 100}{15} = 0.667$$

- B) for an observation that is 85

$$z\ score = \frac{85\ - 100}{15} = -1.0$$

- C) for an observation that is 142

$$z\ score = \frac{142\ - 100}{15} = 2.8$$

Nov. 2021: Warm-UP Answers

2) Using the information from the problem above, what is the value of the observation that has a z-score: a) $z\ score = -2$

$$-2 = \frac{x - 100}{15}$$

$$-30 = x - 100$$

$$\therefore\ x = 70$$

b) $z\ score = 1.58$

$$1.58 = \frac{x - 100}{15}$$

$$23.7 = x - 100$$

$$\therefore\ x = 123.7$$

Standard normal Table

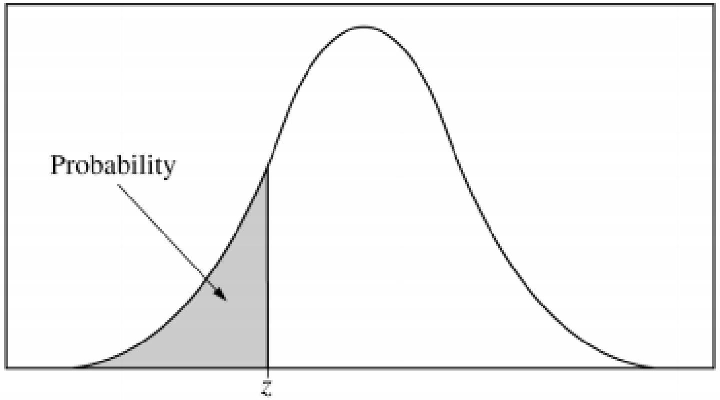

Table entry for *z* is the probability lying below *z*.

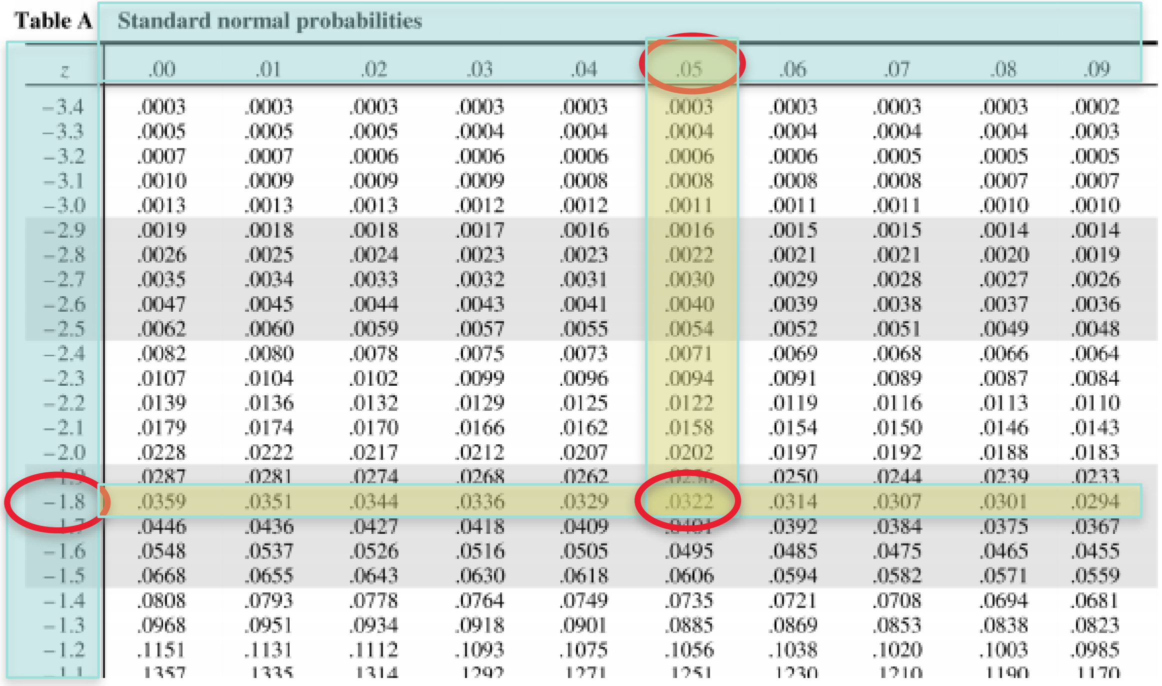

Table A Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	[illegible]	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	1357	1335	1314	1292	1271	1251	1230	1210	1190	1170

What percentile is a *z-score* of -1.85?

a *z-score* of
-1.85 = 0.0322
So about
3.2% or
3rd percentile

Standard normal Table

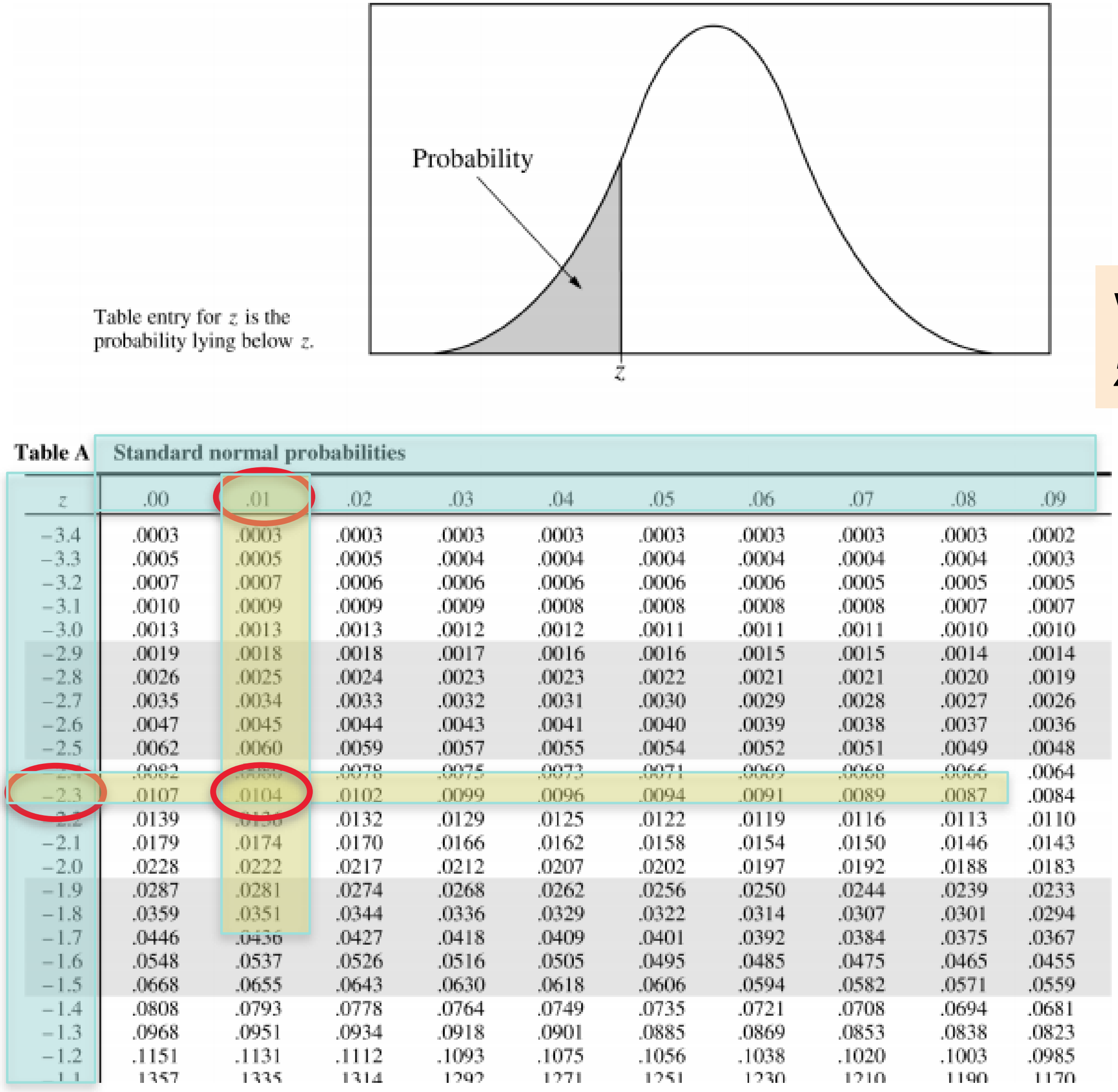

Table entry for *z* is the probability lying below *z*.

Table A Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
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−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170

What percentile is a *z-score* of -2.31?

a *z-score* of
-2.31 = 0.0104
So about
1%
1st percentile

Standard normal Table
Finding the z-score

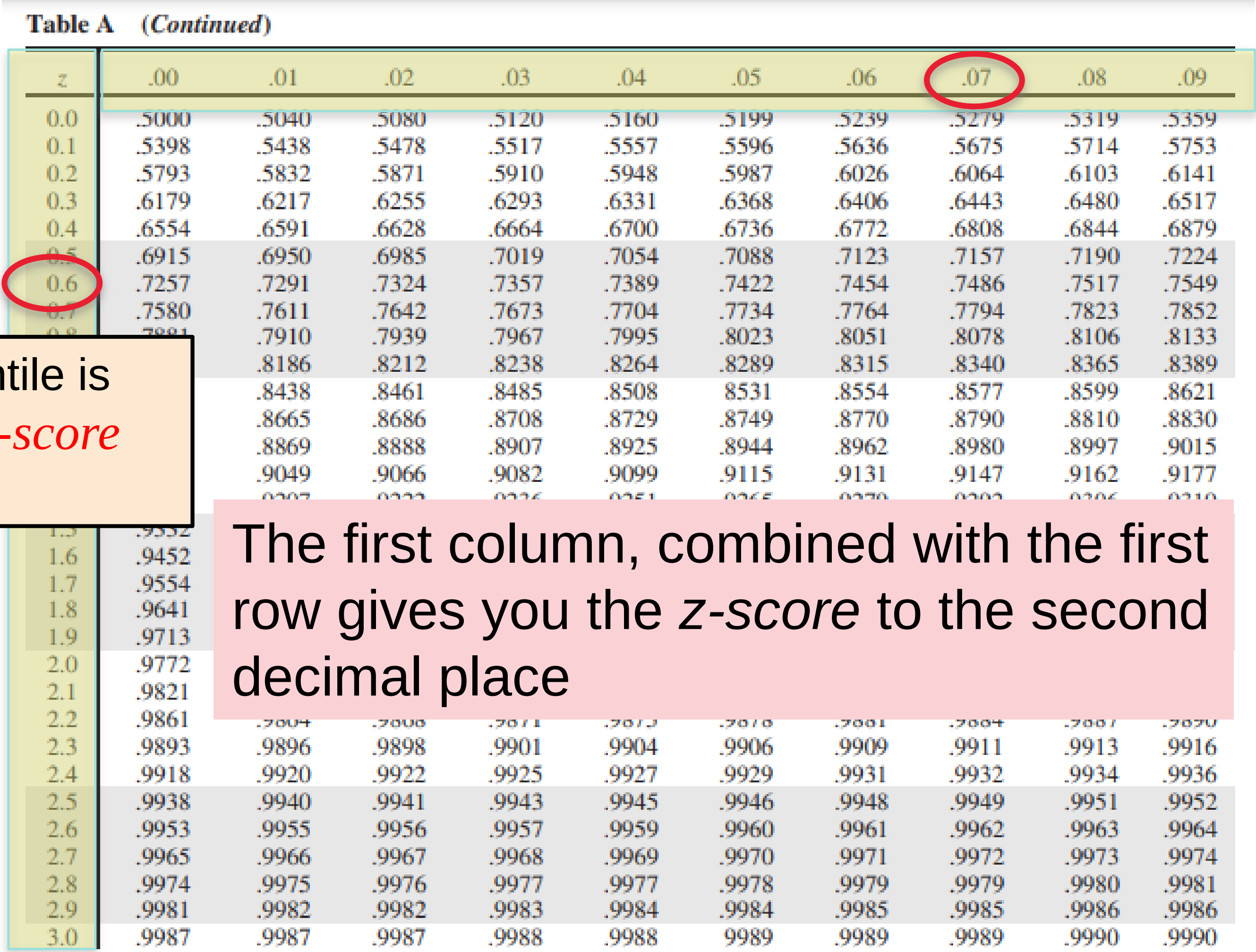


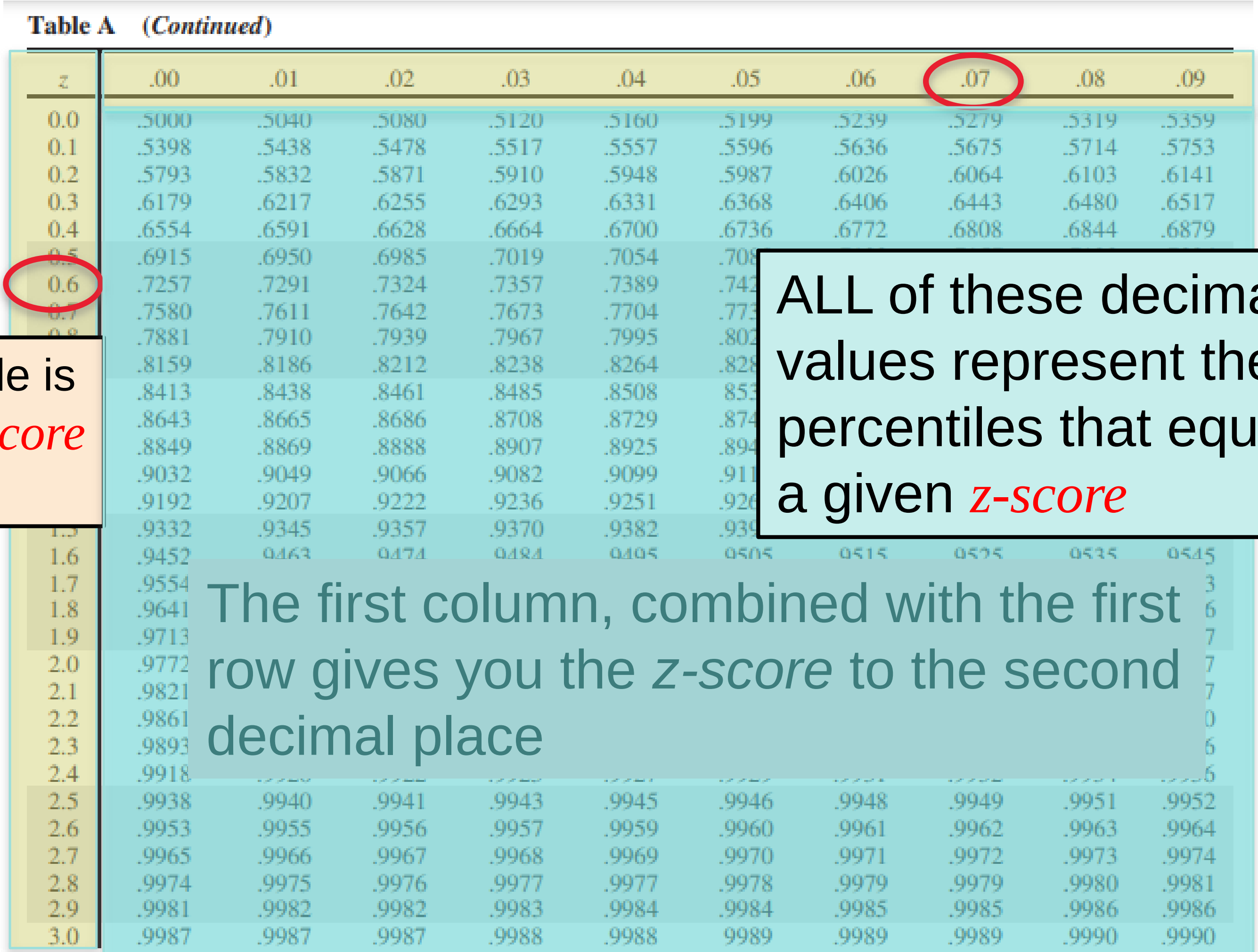
Table A *(Continued)*

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
		.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
		.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
		.8438	.8461	.8485	.8508	8531	.8554	.8577	.8599	.8621
		.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
		.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
		.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.6	.9452									
1.7	.9554									
1.8	.9641									
1.9	.9713									
2.0	.9772									
2.1	.9821									
2.2	.9861									
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	9989	.9989	.9989	.9990	.9990

What percentile is equal to a *z-score* of **0.67**?

The first column, combined with the first row gives you the *z-score* to the second decimal place

Standard normal Table
Finding the *percentile* = z-score



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0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.708	[illegible]	[illegible]	[illegible]	[illegible]
0.6	.7257	.7291	.7324	.7357	.7389	.742	[illegible]	[illegible]	[illegible]	[illegible]
0.7	.7580	.7611	.7642	.7673	.7704	.773	[illegible]	[illegible]	[illegible]	[illegible]
0.8	.7881	.7910	.7939	.7967	.7995	.802	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	.8159	.8186	.8212	.8238	.8264	.828	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	.8413	.8438	.8461	.8485	.8508	.853	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	.8643	.8665	.8686	.8708	.8729	.874	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	.8849	.8869	.8888	.8907	.8925	.894	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	.9032	.9049	.9066	.9082	.9099	.911	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	.9192	.9207	.9222	.9236	.9251	.926	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	.9332	.9345	.9357	.9370	.9382	.939	[illegible]	[illegible]	[illegible]	[illegible]
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1.8	.9641	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1.9	.9713	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2.0	.9772	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2.1	.9821	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2.2	.9861	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2.3	.9893	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2.4	.9918	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	9989	.9989	.9989	.9990	.9990

What percentile is equal to a *z-score* of **0.67**?

ALL of these decimal values represent the percentiles that equal to a given *z-score*

The first column, combined with the first row gives you the *z-score* to the second decimal place

The Standard Normal Table

Because all Normal distributions are the same when we standardize, we can find areas under any Normal curve from a single table.

Definition: **The Standard Normal Table**

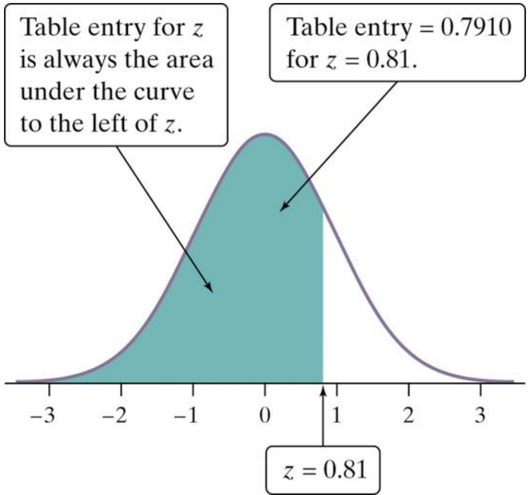
Table A is a table of areas under the standard Normal curve. The table entry for each value *z* is the area under the curve to the left of *z*.

Suppose we want to find the proportion of observations from the standard Normal distribution that are less than 0.81.

We can use Z table (z^*):

Z	**.00**	**.01**	**.02**
0.7	.7580	.7[illegible]1	.7642
0.8	.7881	.7910	.7939
0.9	.8159	.8186	.8212

$P(z < 0.81) = .7910$



Z-score WS practice: $z = \frac{obsev. - mean}{S.D.}$

1. A normal distribution of scores has a standard deviation of 10. Find the z-scores corresponding to each of the following values:

   a) A score that is 20 points above the mean.

   b) A score that is 10 points below the mean.

   c) A score that is 15 points above the mean

   d) A score that is 30 points below the mean.

Z-score WS practice: $z = \frac{obsev. - mean}{S.D.}$

The Welcher Adult Intelligence Test Scale is composed of a number of subtests. On one subtest, the raw scores have **a mean of 35** and a **standard deviation of 6.** Assuming these raw scores form a normal distribution:

a) What number represents the 65th percentile (what number separates the lower 65% of the distribution)?

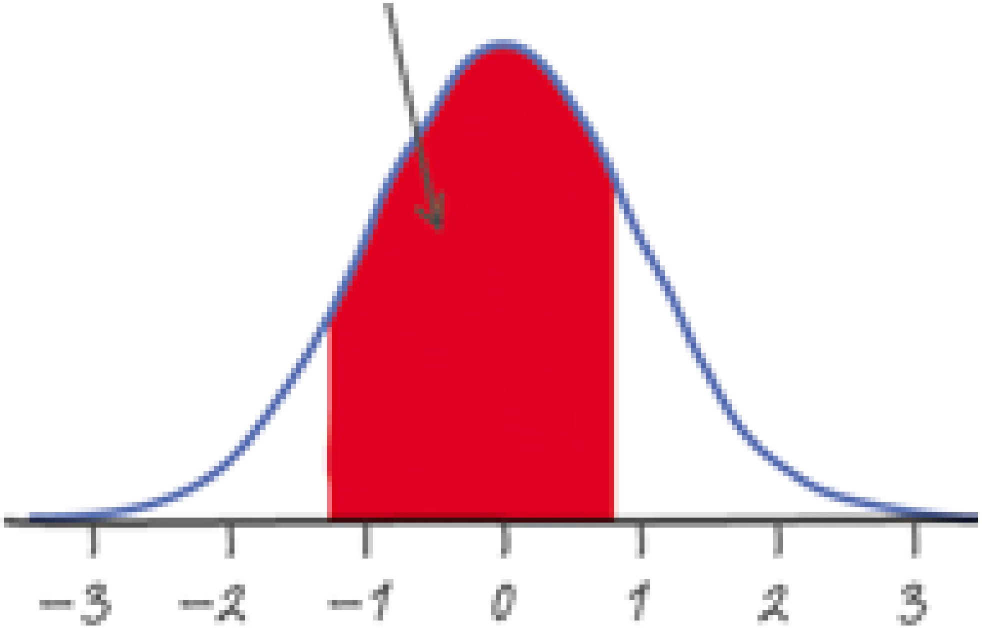
Percentile	Quartile	z-score
65th	$>$ ☐ ***but*** $<$ ☐	**?**

Example

- **Finding Areas Under the Standard Normal Curve**

Find the proportion of observations from the standard Normal distribution that are between -1.25 and 0.81.

What is the area (or proportion of observations)?



Example

Finding Areas Under the Standard Normal Curve

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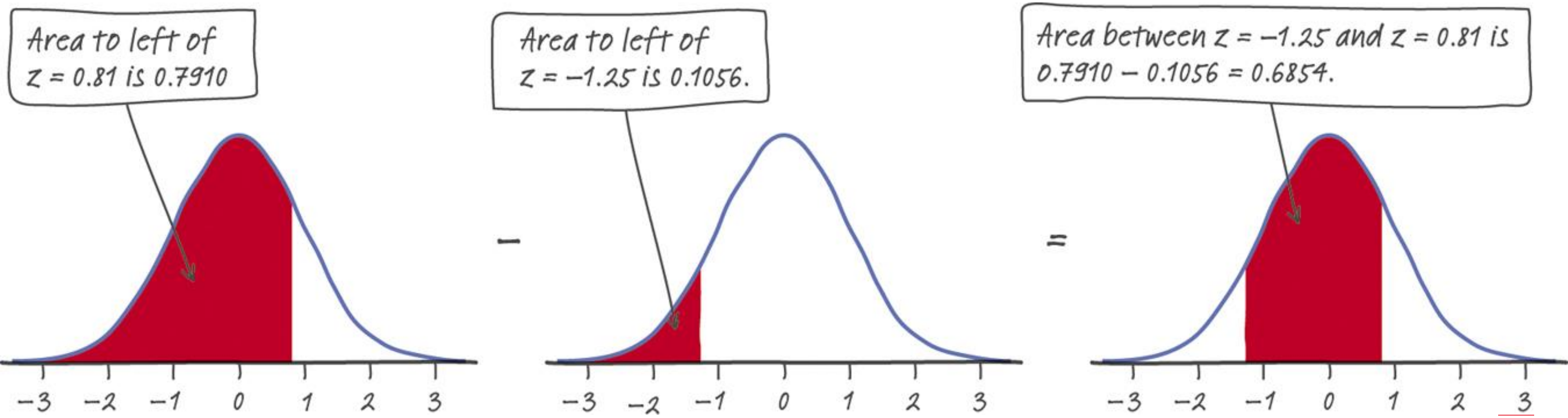
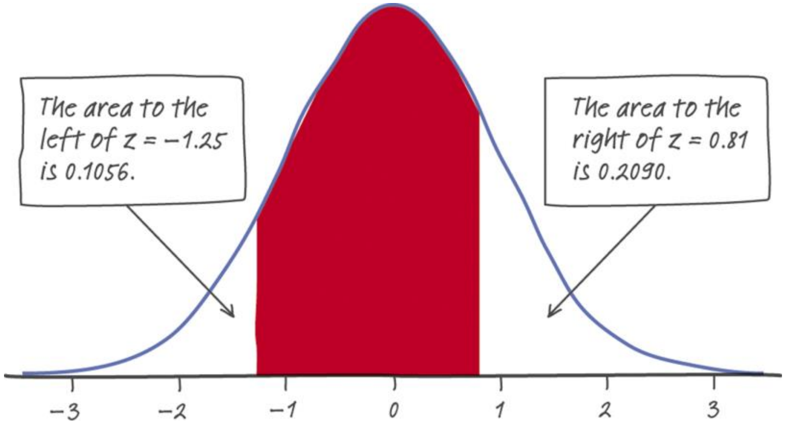


Area to left of $z = 0.81$ is 0.7910

Area to left of $z = -1.25$ is 0.1056.

Area between $z = -1.25$ and $z = 0.81$ is $0.7910 - 0.1056 = 0.6854$.

Can you find the same proportion using a different approach?



The area to the left of $z = -1.25$ is 0.1056.

The area to the right of $z = 0.81$ is 0.2090.

$1 - (0.1056+0.2090) =$

$1 – 0.3146$

= 0.6854

ormal Distributions

Normal Distribution Calculations

How to Solve Problems Involving Normal Distributions

State: Express the problem in terms of the observed variable *x*.

Plan: Draw a picture of the distribution and shade the area of interest under the curve.

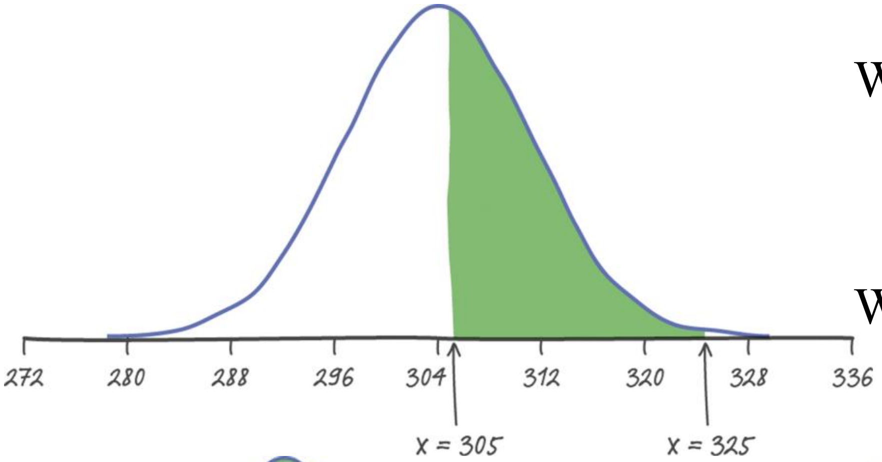
Do: Perform calculations.

- ***Standardize*** *x* to restate the problem in terms of a standard Normal variable *z*.
- ***Use Standard Table*** and the fact that the total area under the curve is 1 to find the required area under the standard Normal curve.

Conclude: Write your conclusion in the context of the problem.

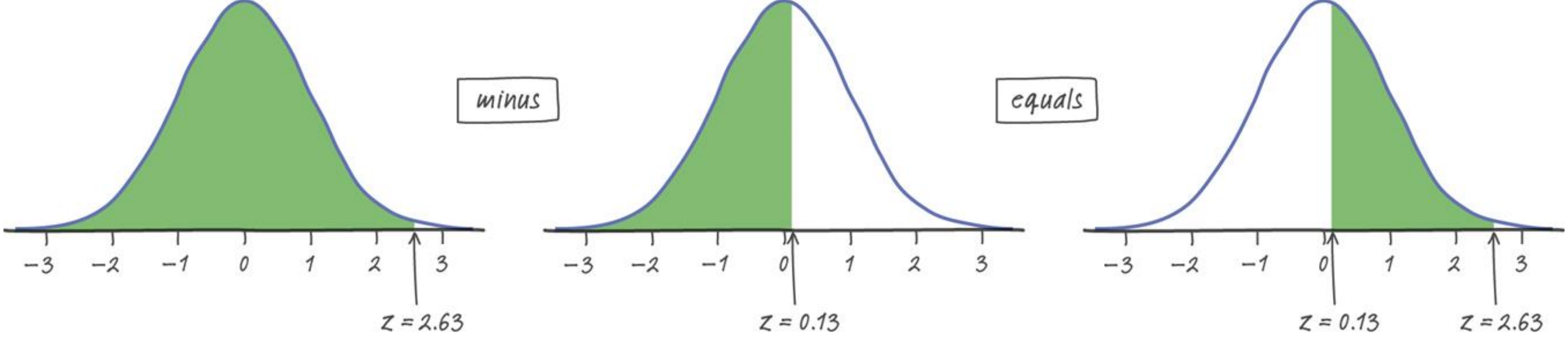
- **Normal Distribution Calculations**

  When Tiger Woods hits his driver, the distance the ball travels can be described by *N*(304, 8). What percent of Tiger's drives travel between 305 and 325 yards?



$$\text{When } x = 305,\ z = \frac{305 - 304}{8} = 0.13$$

$$\text{When } x = 325,\ z = \frac{325 - 304}{8} = 2.63$$



Using Table A, we can find the area to the left of z=2.63 and the area to the left of z=0.13. **0.9957 – 0.5517 = 0.4440**. About **44%** of Tiger's drives travel between 305 and 325 yards.

Assessing Normality

- The Normal distributions provide good models for some distributions of real data. Many statistical inference procedures are based on the *assumption* that the population is approximately Normally distributed. Consequently, we need a strategy for assessing Normality.

✓***Plot the data*.**

- Make a dotplot, stemplot, or histogram and see if the graph is approximately symmetric and bell-shaped.

✓***Check whether the data follow the 68-95-99.7 rule.***

- Count how many observations fall within one, two, and three standard deviations of the mean and check to see if these percents are close to the 68%, 95%, and 99.7% targets for a Normal distribution.

Section 4.5
Normal Distributions

Summary

In this section, we learned that…

- ✓ The **Normal Distributions** are described by a special family of bell-shaped, symmetric density curves called **Normal curves.** The mean μ and standard deviation σ completely specify a Normal distribution *N*(μ,σ). The mean is the center of the curve, and σ is the distance from μ to the change-of-curvature points on either side.
- ✓ All Normal distributions obey the **68-95-99.7 Rule**, which describes what percent of observations lie within one, two, and three standard deviations of the mean.